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ANDREW'S'
PRACTICAL SCHOOL OF ART PERSPECTIVE

FOR THE USE OF
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AND FOR
PRIVATE INSTRUCTION

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—GILL'S SCHOOL SERIES.—

THE
SCHOOL OF ART
SECOND GRADE PERSPECTIVE.

BY

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CHARLES' SCHOOL, PLYMOUTH.

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PREFACE.



HE aim of the following Work is partly defined by its Title. It is a Course of Lessons in PRACTICAL PERSPECTIVE, intended not only for the Examinations of the Science and Art Department, but also for those of Woolwich and Sandhurst; H.M.S. "Britannia;" the Oxford and Cambridge Local Examinations; and all Examinations on that subject.

I have endeavoured to make the Lessons progress by easy steps from the projection of a simple point to that of the most difficult solid; and my aim in the arrangement has been that a Pupil who cannot avail himself of the help of a Teacher, may thoroughly master the subject of Perspective, so as to pass any of the Examinations here mentioned. Whatever the value of this Work may be, I have been most careful throughout its arrangement to produce a manual that should never sink into a mere "CRAM" Book; and, wherever required, I have added many notes and suggestions whilst explaining the Lesson.

In using this Work, I would call the Pupil's attention to the following suggestions:—

- 1.—Never attempt Perspective Drawing without your paper is securely fastened to a drawing-board, and you are provided with a T and set square, and long thin-edged ruler, in addition to the ordinary mathematical instruments.
- 2.—Use the T square for all horizontal lines, and the set square, in conjunction with the T square, for all vertical lines.
- 3.—Take small, common pins, and fix one in the Centre of Vision, Vanishing Points, Measuring Points, &c., and never draw a Line to any of these points unless the ruler rests against that pin. You will then be certain that each parallel line vanishes to one and the same point on the Horizontal Line.
- 4.—Thoroughly master one Lesson, and work the Exercises on that Lesson before proceeding to the next Lesson.

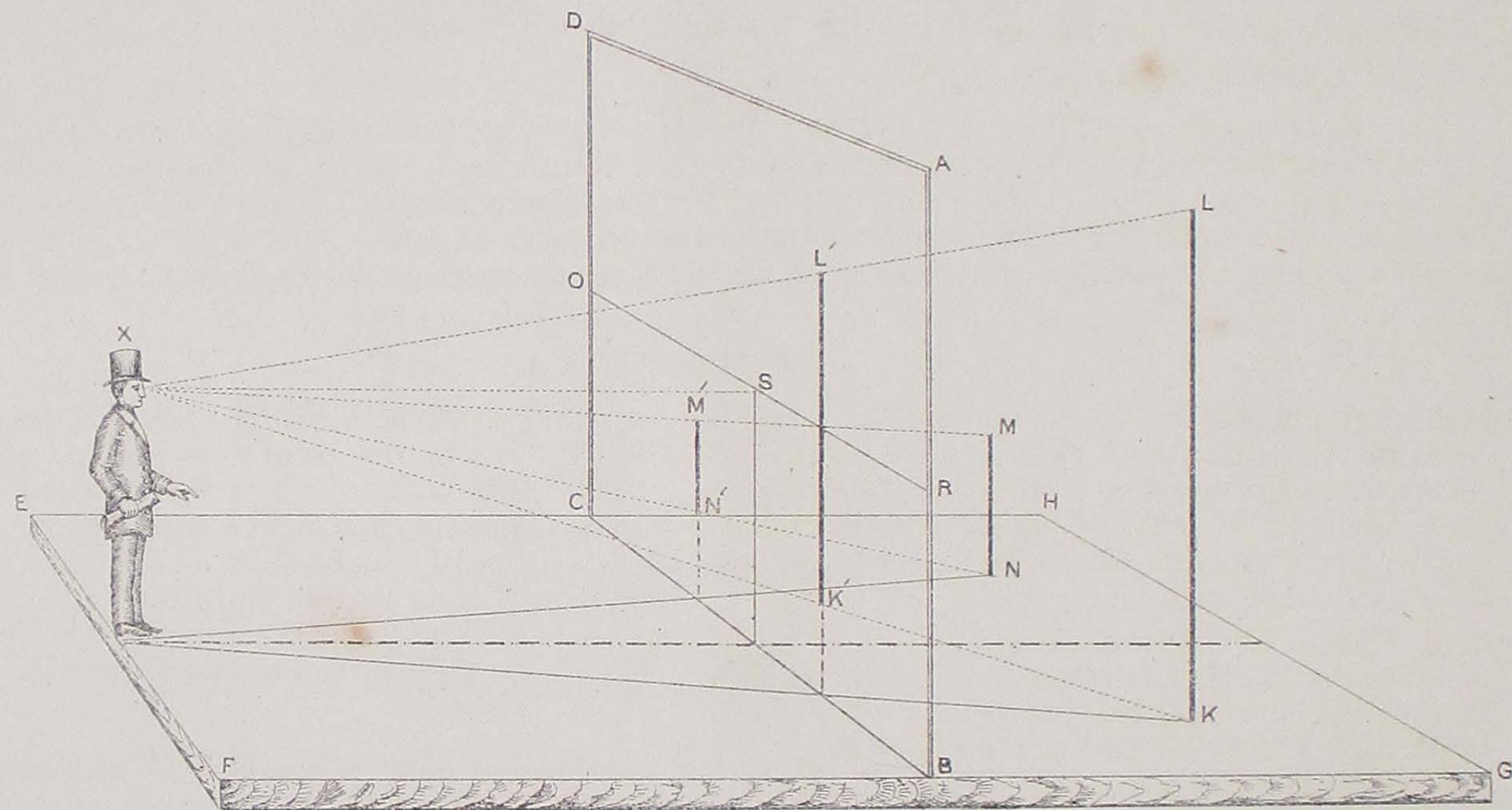
THOMAS NEWTON ANDREWS, F.S.Sc., LONDON.

CHARLES' SCHOOLS, PLYMOUTH, *August 3rd, 1880.*

ABBREVIATIONS USED IN THIS WORK.

P. S.	=	POINT OF SIGHT.
S. P.	=	STATION POINT, THAT IS, THE POSITION OF THE SPECTATOR.
P. D.	=	POINT OF DISTANCE.
V. P.	=	VANISHING POINT.
H. L.	=	HORIZONTAL LINE OR HORIZON.
P. L.	=	PICTURE LINE.
P. P.	=	PICTURE PLANE.
M. P.	=	MEASURING POINT.
C. V.	=	CENTRE OF VISION.
A. V. P.	=	ACCIDENTAL VANISHING POINT.
T. P.	=	TRANSPARENT PLANE.
G. P.	=	GROUND PLANE.
L. D.	=	LINE OF DIRECTION.

LESSON 1.



Andrews' School of Art Second Grade Perspective.

PART I.—LINEAR PERSPECTIVE.

LESSON I. EXPLANATION OF TERMS.

LINEAR PERSPECTIVE may be defined to be the art of representing objects on a given surface in the same forms and relative proportions which they present to the eye, according to their respective distances from the beholder, looking from a given point.

In the accompanying *Figure*, A B C D is a *Transparent Surface*, fixed at right angles to the ground, E F G H ; L K and M N are two sticks, also at right angles to the ground, and behind the transparent surface A B C D ; and the observer at X is looking at those sticks through the transparent surface A B C D . The rays of light proceeding from the sticks to the observer's eye must pass through that transparent surface, and if the points where such rays cut that surface be joined, perspective representations of the sticks are obtained, as shown at L¹ K¹ and M¹ N¹.

DEFINITIONS.

1. **Transparent Plane.** The transparent surface A B C D is called the *Transparent Plane*, and it is on this plane that all perspective representations of objects are obtained. It is the plane cutting the rays of light proceeding from objects behind it to the eye of a person before it.
2. **Ground Plane.** The plane surface E F G H representing the Earth's surface is called the *Ground Plane*, and the line B C where the planes A B C D and E F G H intersect each other is the *Ground or Picture Line*.
3. **Horizon or Horizontal Line.** The Line O R drawn across the transparent plane at the height of the man's eye above the ground is the *Horizon or Horizontal Line*. The point S in that line and directly opposite his eye is the *Point of Sight or Centre of Vision*.
4. **Line of Direction.** The line X S from the spectator's eye to the point of sight is called the *Line of Direction*, i.e., it is the direction the spectator is looking ; and on looking at the figure, it will be seen that the stick L K is on the right of this line, whilst M N is on the left ; hence, in the subsequent Lessons, objects are put in perspective sometimes on the left, and at other times on the right of the spectator.
5. **Station Point** is the point X, before the transparent plane, where the spectator stands when looking at the objects behind the transparent plane.
6. **Points of Distance** are the points, measured from the point of sight, on the horizon, and at the same distance from that point that the spectator is from it. (See Lesson II.)

LESSON II.

PLATE II

LESSON II.

THE POSITION OF THE LINES AND PLANES IN PRACTICAL WORKING.

The figure in this Lesson is the same as in Lesson I., the whole model being turned round through 90 degrees. The spectator is at X , and the transparent plane $A B C D$ is hinged at 1 and 2, so that it may either be placed vertically as in Lesson I., or horizontally, as shown in this Lesson.

When the spectator looks at anything, such as the surrounding country, all that his eye can take in at one view is included within the base of a *Cone of Rays*, having an angle of 60° at its apex, and no object falling without this angle can be seen without moving the eye. In the diagram, the observer is at the *station point* X , and the horizon is drawn across the transparent plane $A B C D$ (as in Lesson I.), the height of his eye above the ground plane. Make the angle $R X O$, 60° , cutting the horizon in O and R . Then $O R$ is the diameter of the base of a cone of rays that will include all objects seen by an observer, stationed at X , at one view. From the P. S. or C.V.

describe the circle with half of $O R$ as radius, and from the same point with the distance of the observer from it (C. V. to X), describe the semicircle cutting the horizon in P. D.¹ and P. D.². These are the Points of Distance to which points all lines inclined at 45° to the transparent plane will vanish.

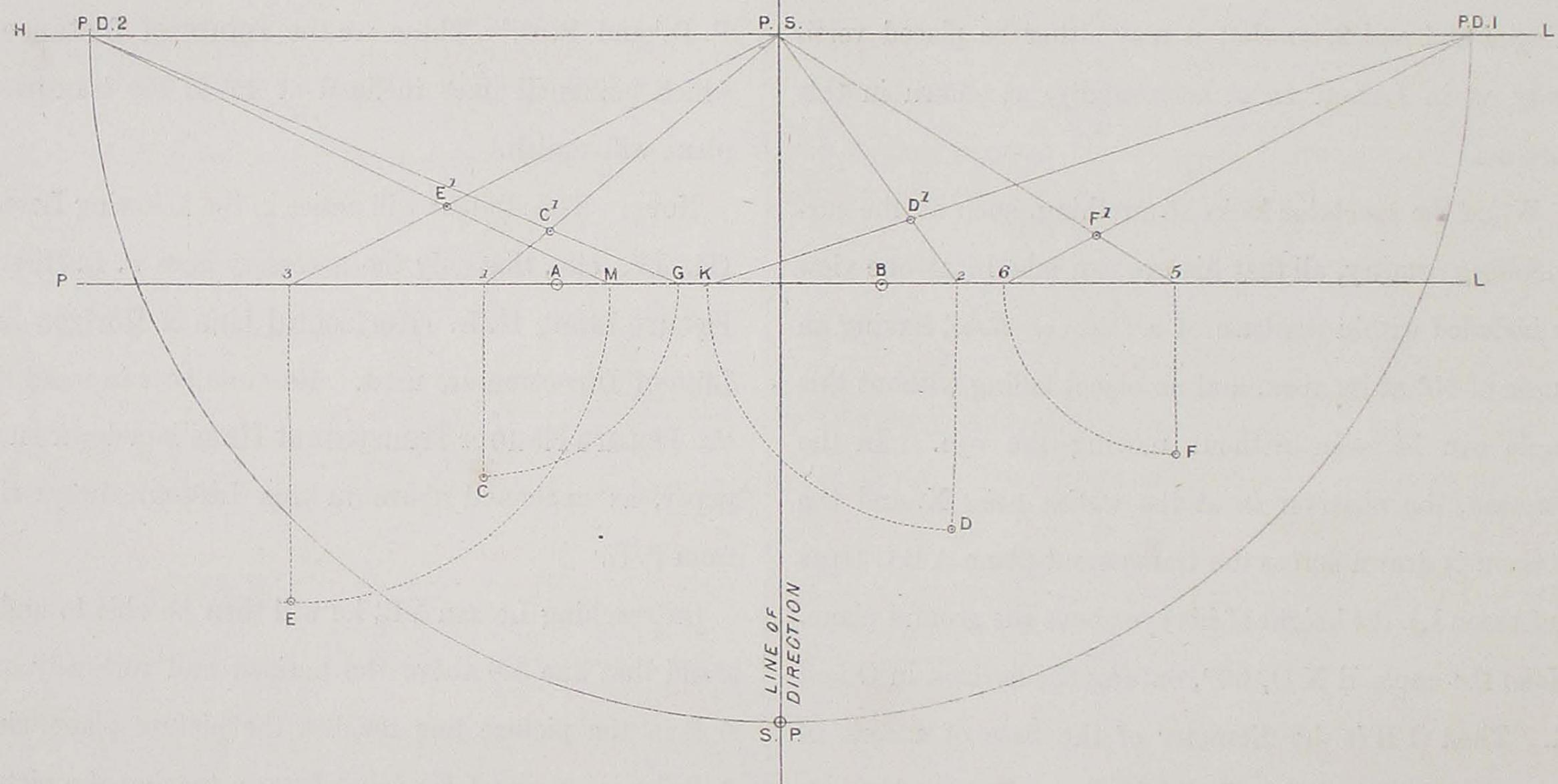
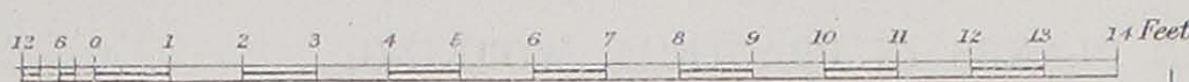
NOTE.—The student will notice in the following Lessons III., IV., etc., that only the necessary lines P. L. (that is, Picture Line), H. L. (Horizontal Line or Horizon) and Line of Direction are used. He must bear in mind that the Picture Plane or Transparent Plane laid down on his paper, as explained above in this Lesson, always rises from P. L.

On reaching Lesson VI., he will then be able to understand that line 5-6 above the horizon and vertically over $A B$ in the picture line touches the picture plane, since $A B$ does; so also $J K$ of that Lesson touches the picture plane since J does, and K is vertically over it.

LESSON III

PLATE III

SCALE 48



LESSON III. ON THE PERSPECTIVE PROJECTION OF POINTS.

The student, having thoroughly mastered the two former lessons, may now commence to put points in and take them out of perspective, under the following conditions (*Plate 3*) :—

The observer is to stand 14 ft. from the PICTURE PLANE (Transparent Plane), the height of his eye is 5 ft. above the ground, and the scale of the drawing is $\frac{1}{8}$, that is, $\frac{1}{4}$ of an inch represents a foot.

Fix a sheet of paper to the drawing board and draw the horizontal line H. L. Fix the point P. S., and draw the line of direction at right angles to H. L. The station point S. P. is in this line 14 ft. from the P. S. (point of sight). Describe the semicircle, from P. S., with 14 ft. radius, cutting the horizontal line in P. D.¹ and P. D.² (points of distance), and draw the picture or ground line P. L. 5 ft. below the horizontal line and parallel to it.

Problem 1.—Find a point A in the picture plane, and on the ground 4 ft. 6 in. to the spectator's left.

Take the distance 4 ft. 6 in. in the compasses, and from the line of direction cut the picture line in point A on the observer's left, that is, to the left of the line of direction.

Problem 2.—Find a point B in the picture plane, and on the ground 2 ft. to the right of an observer at S. P.

Take the distance 2 ft. in the compasses and cut the picture line in B, 2 ft. from the line of direction, on the right.

Problem 3.—Find a point C on the ground, 6 ft. to the observer's left and 4 ft. *within the picture*, that is, the point C is that distance behind the transparent plane.

Take the distance, 6 ft., and cut the picture line in point 1 to the left of line of direction, draw the perpendicular dotted line 1 C—4 ft. long (the distance the point is within the picture). Join point 1 to P. S., and from 1 as centre and 1 C (4 ft.) as radius describe the arc C G. Join G to P. D.² cutting the line 1—P. S. in C¹, then C¹ is the point C in perspective under the given conditions.

Problem 4.—Find a point D on the ground, 3 ft. 6 in. to the observer's right and 5 ft. within the picture.

Take the distance 3 ft. 6 in. and cut the picture line in point 2. Draw the perpendicular dotted line 2 D, make 2 D, 5 ft. long, and describe the arc D K. Join 2 to P. S. and K to P. D.¹ cutting each other in D¹, which is the point required.

Problem 5.—Given a point E¹ on the ground in the perspective to find its distance to the left and within the picture.

This Problem is the reverse of the former ones, the point being given in perspective to take it out. Draw lines through the point E¹ from P. S. and P. D.² to cut the picture line in points 3 and M. Draw the perpendicular line 3 E, and from 3 describe the arc M E with radius 3 M. E is the point out of perspective, and measured by scale it is found to be 10 ft. to the left of the line of direction; and the perpendicular 3 E, is 6 ft. 6 in. long, so that the point E¹ is 10 ft. to the left and 6 ft. 6 in. within the picture.

Problem 6.—Given a point F¹ in perspective on the ground to find its distance to the right, and within the picture.

Proceed as in Problem 5, by drawing the lines from P. S. and P. D.¹ through the point F¹, cutting the picture line in points 5 and 6. It will be found on measurement that the point F¹ is 8 ft. to the right, and 3 ft. 6 in. within the picture.

EXERCISES ON LESSON III.

Find the perspective projections of the following points on the ground :—N. 3 ft. to the left and 6 ft. 6 in. within the picture. O. 8 ft. to the right and *in the* picture plane. P. 5 ft. 9 in. to the right and 4 ft. 6 in. within the picture. Also, fix any points Q and R, between the horizontal and picture lines and take them out of perspective.

NOTE.—(1) *All distances* must **ALWAYS** be measured on the Picture Line to the right or left of the line of direction.

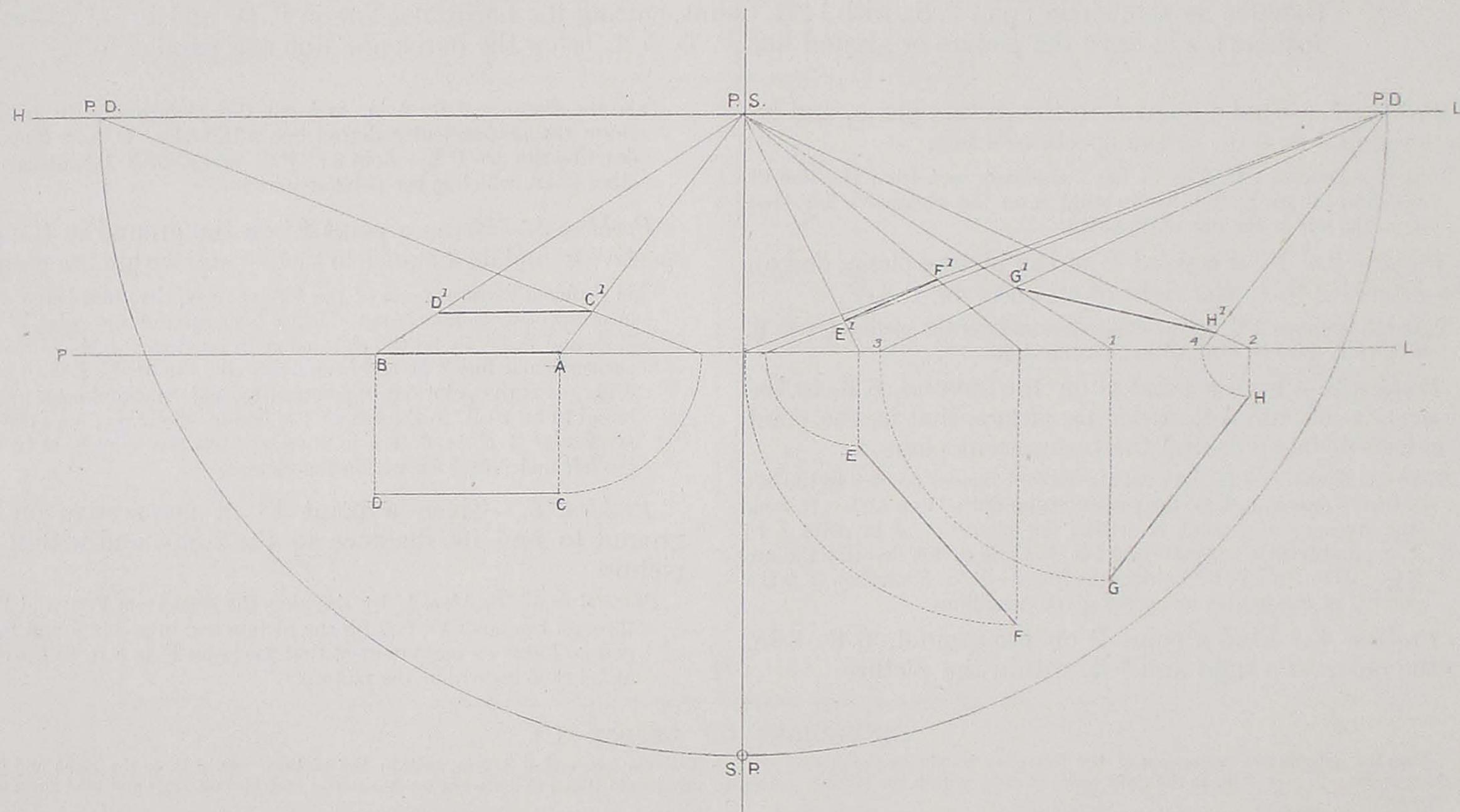
(2) The points C, D, E, F, below the Picture Line, are the **Plans** of the corresponding points in Perspective.

(3) In Taking Points out of Perspective, always drop the Perpendicular Line from the line drawn through the required point, from the **Point of Sight**, e.g. 3 E, 5 F, etc.

LESSON IV.

PLATE IV

SCALE $\frac{1}{48}$



LESSON IV.

ON THE PERSPECTIVE PROJECTION OF HORIZONTAL LINES.

Given the point of sight (P.S.) Distance of spectator (S. P.) 14 ft. Height of eye, 5 ft., to determine the following Problems :—

Problem 1.—A line A B, 4 ft. long, lies in the picture and ground planes, with the point A 4 ft. to the left. Find it.

Having drawn the H. L., P. L., and line of direction, as explained in Lesson III., set off the point A on the picture line 4 ft. to the left of the line of direction, and B 4 ft. further to the left on that line; then A B is the line required, as such a line must be on the intersecting line of the picture and ground planes. Look at *Plate 1* and this will be readily seen.

Problem 2.—A line C D, 4 feet long, lies on the ground and is parallel to the picture plane. The point C being 4 ft. to the left and 3 feet within the picture.

Below the picture line and parallel to it, place the line C D so that the point C is 4 ft. to the left of the *line of direction*, and the line C D 3 ft. below the *picture line*. Place the point C in perspective as in Lesson III. (it will be the point C¹) and draw C¹ D¹ parallel to P. L. the picture line; the point D¹ is the point D in perspective, and C¹ D¹ is the line required.

Problem 3.—Given E F, the plan of a line, inclined to the picture plane, to place it in perspective.

Place the two points E and F in perspective, as explained in Lesson III. It will be E¹, F¹, join E¹ F¹ for the perspective projection of the line E F.

Problem 4.—A line G¹ H¹ is given in perspective. Find : (1) The true length of the line. (2) Its distance to the right of spectator. (3) The distance of each end within the picture.

Take the points G¹ and H¹ out of perspective (Lesson III.) by drawing lines through these points from the P. S. and P. D. Drop perpendicular lines from the picture line at 1 and 2, and with radius 1 3 cut the line 1 G in G (the point G¹ out of perspective), and with radius 2 4 cut the line 2 H in H (the point H¹ out of perspective). The line G H is the real length of G¹ H¹; measured to scale $\frac{1}{48}$ it is 5 ft. long. The line 1 G is the distance of G¹ within the picture (5 ft.) and 2 H of H¹ within the picture (1 ft.) The distance of the points 1 and 2 on the picture line from the line of direction is the distance of G¹ and H¹ to the right of the spectator.

Problem 5.—Find the true distance from the point C¹ to G¹.

First take each point out of perspective as explained (Lesson III.) This distance, when taken out of perspective, is the true distance of the points C and G from each other: that is, they are plans of points C¹ and G¹, which, measured to scale $\frac{1}{48}$, is 12 ft. C is the point C¹ and G the point G¹, taken out of perspective, and consequently in their true position. (From C to G is 12 ft.)

EXERCISES ON LESSON IV.

Place the following lines in Perspective—(1) A line A B, 3 ft. 6 in. long, parallel to the picture plane and lying on the ground, with point A 2 ft. 6 in. to the left and 3 ft. within the picture. (2) A line E F inclined to the picture plane, as in Problem 3 above, lying on the ground, with the point E 2 ft. to the right and 1 ft. within, and point F 6 ft. to the right and 4 ft. within. (3) Draw a line, 3 ft. long, by scale, between the H. L. and P. L. on the left, and not parallel to the picture line; suppose it to be a stick lying on the ground; find (1) its true length, (2) the distance of each end to the left, and (3) the distance within the picture.

NOTES.—(1) The true length of a line given in perspective on the ground is the length of the plan of that line.

(2) The true distance between any two points given in perspective on the ground is the distance that those points are from each other when taken out of perspective.

(3) No lines or points can be measured in perspective; they must first be taken out of perspective.

LESSON V.

PLATE V.

SCALE $\frac{1}{48}$

LESSON V.

ON THE PERSPECTIVE PROJECTION OF VERTICAL LINES.

Having drawn the *horizontal* and *picture lines*, etc., as before, let it be required to solve the following Problems:—

Problem 1.—A vertical line A B, 10 ft. high, is in the picture plane, with its end A resting on the ground plane 8 ft. to the left.

Find the point A in the picture line 8 ft. to the left (Lesson III., Problem 1.) Set up the perpendicular line A B 10 ft. high from the scale $\frac{1}{8}$ (Plate III.) A B is the required line.

Problem 2.—A vertical line C D, 4 ft. high, has its point C fixed on the ground 4 ft. to the left and 5 ft. within the picture.

Set off on the P. L. the point C¹ 4 ft. to the left. Join C¹ to P S, set back 5 ft. from C¹ on the P. L. to point 1, and join to P. D.², cutting the line to P. S. in C². This point is where the point C enters the ground, and is found as in Problem 3 of Lesson III. You will notice here that although the plan of the point is shown at C below the picture line, yet the point C² can be found without obtaining that plan, by setting off C¹ 1 on P. L. Set up the perpendicular line C¹ D on the picture line, 4 ft. high. Join D to P. S. and draw C² D¹ perpendicular to meet that line; then C² D¹ is the line required.

Problem 3.—A vertical line E F, 8 ft. high, is fixed in the ground 9 ft. to the right and 3 ft. within the picture.

The point E² is obtained as in Problem 2, thus: Set back E¹ 2 equal to 3 ft. on P. L., join to P. D.¹, cutting the line to P. S. in E². Set up E¹ F—8 ft. high, join F to P. S., and draw E² F¹ perpendicular, as in the last Problem. E² F¹ is the line required.

Problem 4.—A bird H¹ is given, in perspective, 4 ft. to the right. To find its height above the ground.

First find the point 3 on P. L. 4 ft. to the right (Lesson III). Set up a perpendicular line 3 H to meet a line from P. S. through the point H¹ where the bird is; 3 H is its height above the ground, which measure to the scale.

Problem 5.—A bird K¹ is given in perspective, 11 ft. high. Find its distance to the left of the spectator.

From P. S. draw a line through K¹ until a perpendicular line K L meeting this line, and perpendicular to P. L., is 11 ft. long. The distance of L from the line of direction (6 ft.) is the distance to the left of the spectator.

Problem 6.—Given the point C² on the ground and the line E² F¹, to find: (1) The distance from C² to E². (2) The height of the line E² F¹. (3) The length of a string to reach from C² on the ground to the top F¹ of the line E² F¹.

Take the points C² and E² out of perspective (Lesson III., Problems 5 and 6) as at E and C below the picture line. E C is their true distance apart. The height of E² F¹ is found thus: From P. S. draw lines through E² and F¹, the former meeting the picture plane in E¹. Erect a perpendicular line E¹ F from E¹ to meet the line coming from P S through F¹; then E¹ F is the true height of E² F¹ (this is the reverse of Problem 3 of this Lesson). The length of the string is found by making the angle C E F¹ 90°. Make E F¹ equal to the height of the line E² F¹ when taken out of perspective (i.e., E¹ F, 8 ft.) and the hypotenuse C F¹ is the length required (15 ft. 4 in).

EXERCISES ON LESSON V.

- (1) Find the line X Y, 7 ft. high, perpendicular to the ground, with its point Y fixed in the ground 5 ft. 6 in. to the left and 4 ft. within the picture.
- (2) A bird is 10 ft. above the ground and 7 ft. within the picture and 6 ft. on my right, show it in perspective.
- (3) Find the length of the line that will reach from the point Y on the ground to this bird.

NOTES.—(1) All heights are always measured from the picture line.

(2) The length of a line to reach from any point on the ground to any other point above the ground is the hypotenuse of a right angled triangle, of which the base is the true distance on the ground of those points apart, and the perpendicular the height of the one point above the ground.

(3) In Problem 4 above—If a perpendicular line be drawn from H¹ to meet the line from 3 to P. S., the point of intersection will be directly under the bird and on the ground, which point taken out of perspective will give the distance the bird is within the picture; so also a perpendicular line from K¹ meeting the line from L to P. S. (Problem 5) will be a similar point for the bird K¹.

LESSON VI.

PLATE VI.

SCALE 1:48

LESSON VI.

ON THE PERSPECTIVE PROJECTION OF THE SQUARE AND RECTANGLE.—(*Height of Eye, 5 ft. ; Distance 14 ft., as in preceding Lessons.*)

Problem 1.—A square of 4 ft. sides lies on the ground with one side touching the picture plane, its nearest angle being 2 ft. to the left. Show it in perspective.

Find the H. L. and P. L., &c., as explained before.

Set off the side A B, 4 ft. long, on the picture line so that point A is 2 ft. to the left. (Problem 1. Lesson IV.) Join A and B to P. S., and draw a line from A to P. D.¹, cutting the line from B in C: then A C is a diagonal of the square. Draw C D parallel to A B. A B C D is the square in perspective.

Note.—By drawing a line from D to P. D.¹ a point F is obtained, and E F drawn parallel to C D gives the square C D E F in perspective under the following conditions:—C D is 4 ft. within the picture, the point D being 2 ft. to the left. The line C D would be obtained in working such a problem, as in Problem 2, Lesson IV. The diagonal from C to P. D.² may be used instead of the one from D to P. D.¹: then point E would be obtained.

Problem 2.—A square of 4 ft. sides is at right angles to the ground and picture planes, one side touching the picture plane 11 ft. to the left. Show it in perspective.

Find the point J—11 ft. to the left (Lesson III.) on the picture line. Set up the perpendicular line J K 4 ft. high. Join J and K to P. S., and join from K to station point S. P., cutting the former line in L. Erect the perpendicular line L M. J K L M is the required square, and K L is one diagonal.

Note.—By drawing the line from M to S. P. the square L M N O is obtained, and by drawing from O to S. P. the square N O P Q, etc. The second square L M N O fulfils the following conditions:—A square at right angles to ground and picture planes, having its nearest angle 11 ft. to the left and 4 ft. within the picture plane. The dotted lines J 1 and arc 1 2 show how L may be found, as in Problem 3, Lesson III.

Problem 3.—A rectangle 4 ft. by 6 ft. lies on the ground with its longer sides perpendicular to the picture plane,

its nearest angle being 2 ft. to the right and 1 ft. within the picture plane. Put it in perspective.

Draw the plan below P. L. so that point R is 2 ft. to the right and 1 ft. below P. L. Construct the rectangle R T U S so that R T, a longer side, is perpendicular to P. L.

Produce U S to 3 and R T to 4, and join the points 3 and 4 to P. S. Cut the line from 3 to P. S. for points S¹ and U¹, as in Lesson III., and complete the figure in perspective by the parallel lines S¹ R¹ and U¹ T¹.

Problem 4.—On one end, U¹ T¹, of the rectangle, Problem 3 (the farthest from the picture plane), a square stands at right angles to the ground and parallel to the picture plane. Show it in perspective.

The height is obtained thus: Set up 3.4 on the picture line 4 ft. high, and join to P. S. From U¹ erect the perpendicular to cut the line, and complete the square.

Problem 5.—The two squares of Problem 1 A B C D and C D E F are raised parallel to the ground, and a side of one kept touching the picture plane, with its nearest angle 2 ft. to the left and 10 ft. high. Show their perspective projections.

Set up the height A 5—10 ft. Draw 5-6 parallel to P. L., making 5-6 equal A B. Join 5 and 6 to P. S., and complete by marking the points 7, 8, 9, &c., perpendicularly above D, E, C, &c., or the squares may be completed after the side 5-6 is obtained by drawing the diagonals to P. D.¹, cutting the line from 5 and 6 in points 9 and 10, as was explained in Problem 1 of this Lesson. (See Note, Lesson II.)

EXERCISES ON LESSON VI.

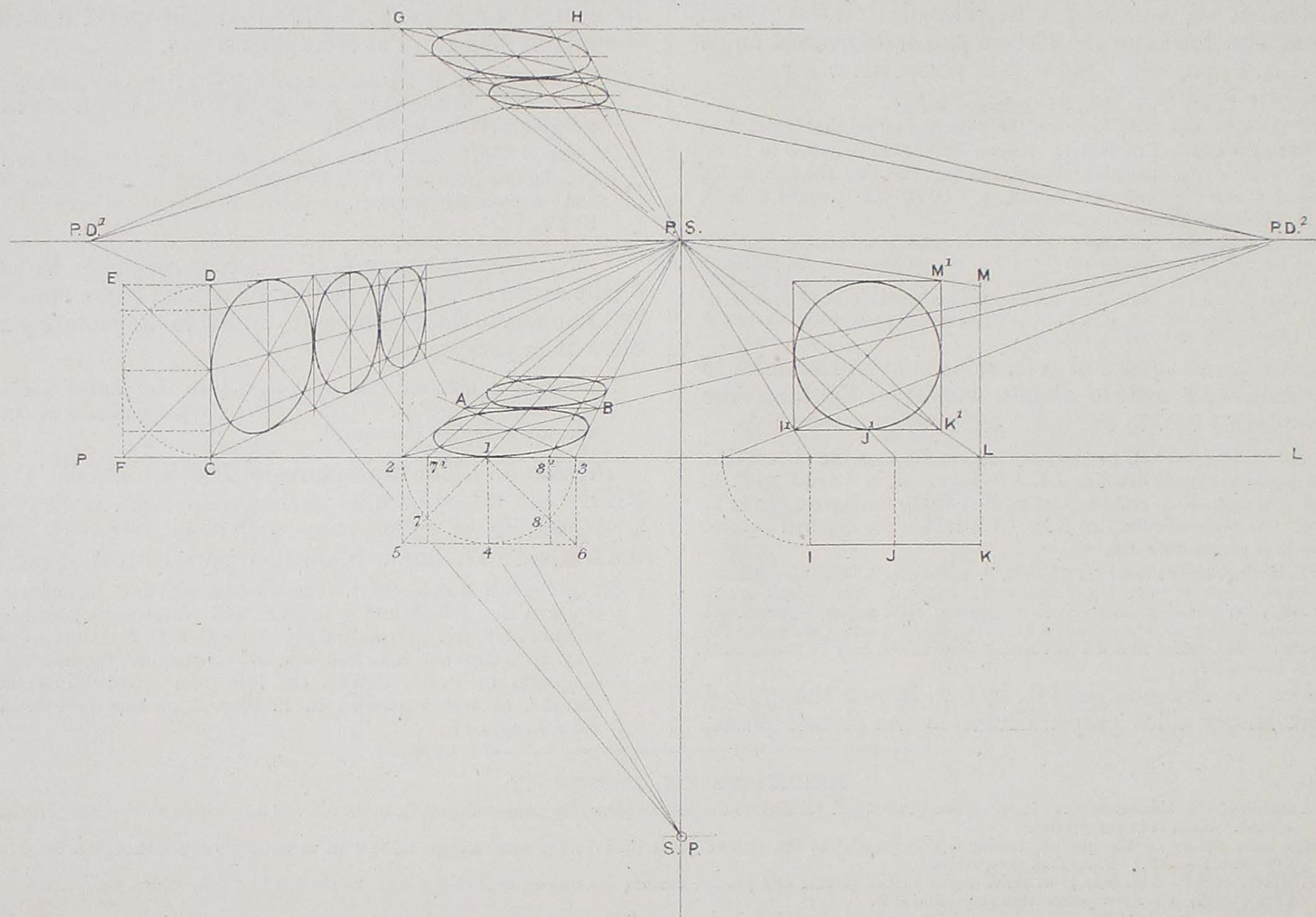
- (1) A square of 4 ft. sides stands vertically on the ground and parallel to the picture plane, its nearest angle 4 ft. to the left and 3 ft. within the picture (Problem 4, above). Show it in perspective.
- (2) The same square on the ground, having a side parallel to the picture plane (P. L.); the nearest angle to be 4 ft. to the right and 2 ft. within the picture. (Problem 1—*Note.*) Place it in perspective.
- (3) A square of 4 ft. 6 in. side is at right angles to the picture and ground planes; its nearest angle being 3 ft. to the left and 5 ft. within the picture plane. (Problem 2.) Give the perspective projection of it.

Notes.—(1) The diagonals of a square lying on the ground with its side parallel—as C D (or, perpendicular, as A D)—to the picture plane *will always* meet the P. D.
(2) The diagonal of a square at right angles to both planes (picture plane and ground plane)—as the squares K L, M N, and O P, *will always* meet the S. P.
(3) A square at right angles to the ground plane and parallel to the picture plane *is always* a perfect square when put in perspective, as in Problem 4.

SCALE $\frac{1}{48}$

LESSON VII.

PLATE VII



LESSON VII.

ON THE PERSPECTIVE PROJECTION OF THE CIRCLE.

(*Distance, height of eye, etc., as in last Lesson.*)

Points of Distance.—In this Lesson the semicircle (described from P. S. with S. P. as radius) is left out purposely; it seldom being drawn in practice. The points P. D.¹ and P. D.² are at once set off from P. S. with the distance of station point.

Problem 1.—A circle of 4 ft. diameter lies on the ground and touches the picture plane. Its centre is 4 ft. 6 in. to the left. Place it in perspective. (Scale $\frac{1}{8}$ in.)

This circle will appear as an ellipse when put in perspective, and is found thus. Set off on the P. L. the point 1, 4 ft. 6 in. to the left. Join to P. S. The centre of the circle will be on this line, and the circle will touch P. L. (i.e., the picture plane, as the Problem requires, in point 1). From point 1 as centre and 2 ft. radius describe the dotted semicircle 2, 3, 4. Surround it with the rectangle 2, 5, 6, 3. Join 1, 6 and 1, 5, cutting the semicircle in 7 and 8. Join points 2, 7¹, 1, 8¹, 3, to P. S. Draw the diagonals 3 A and 2 B to P. D.¹ and P. D.²; join A B; then A B 2-3 is the square surrounding the circle, and through the points where the lines from 7¹ and 8¹ cut the diagonal draw the ellipse by hand, touching the sides of the square.

NOTE.—By drawing the diagonals from A and B the second circle can be placed in perspective, as was the second square in Lesson VI., Problem 1.

Problem 2.—A circle of 4 ft. diameter stands vertically to the ground and picture planes, and touches the picture plane 11 ft. to the left.

Find the point C—11 ft. to the left on P. L., make C D—4 ft. high and complete the square surrounding the circle by drawing the diagonal

from D to S. P. (Lesson VI., Problem 2). On C D describe the dotted semicircle and enclose it by the rectangle C D E F to obtain points as in Problem 1. Join, as shown, to P. S., cutting the diagonals, and describe the ellipse through the points by hand.

NOTE.—The second and third circles are similarly obtained. (See the squares in Lesson VI.)

Problem 3.—The two circles in Problem 1 are shown raised 10 ft. high, parallel to the ground. The points G and H are vertically over 2 and 3, as the squares in Problem 5, Lesson VI.

Each of the points on 2 and 3 is marked on G H and joined as shown, the ellipse being described by hand.

Problem 4.—A circle of 4 ft. diameter stands at right angles to the ground and parallel to the picture plane, its centre being 5 ft. to the right and 2 ft. within the P. P.

The line I J K shows the plan of this circle, the centre J being 5 ft. to the right and 2 ft. below P. L. (Lesson IV., Problem 2.) Place it in perspective I¹ J¹ K¹. Set up L M—4 ft. high, join to P. S., and draw K¹ M¹ perpendicular. I¹ K¹ and K¹ M¹ are two sides of the square surrounding the circle. (Lesson VI., Problem 4.) Complete the square, and inscribe the circle with the compasses.

EXERCISES ON LESSON VII.

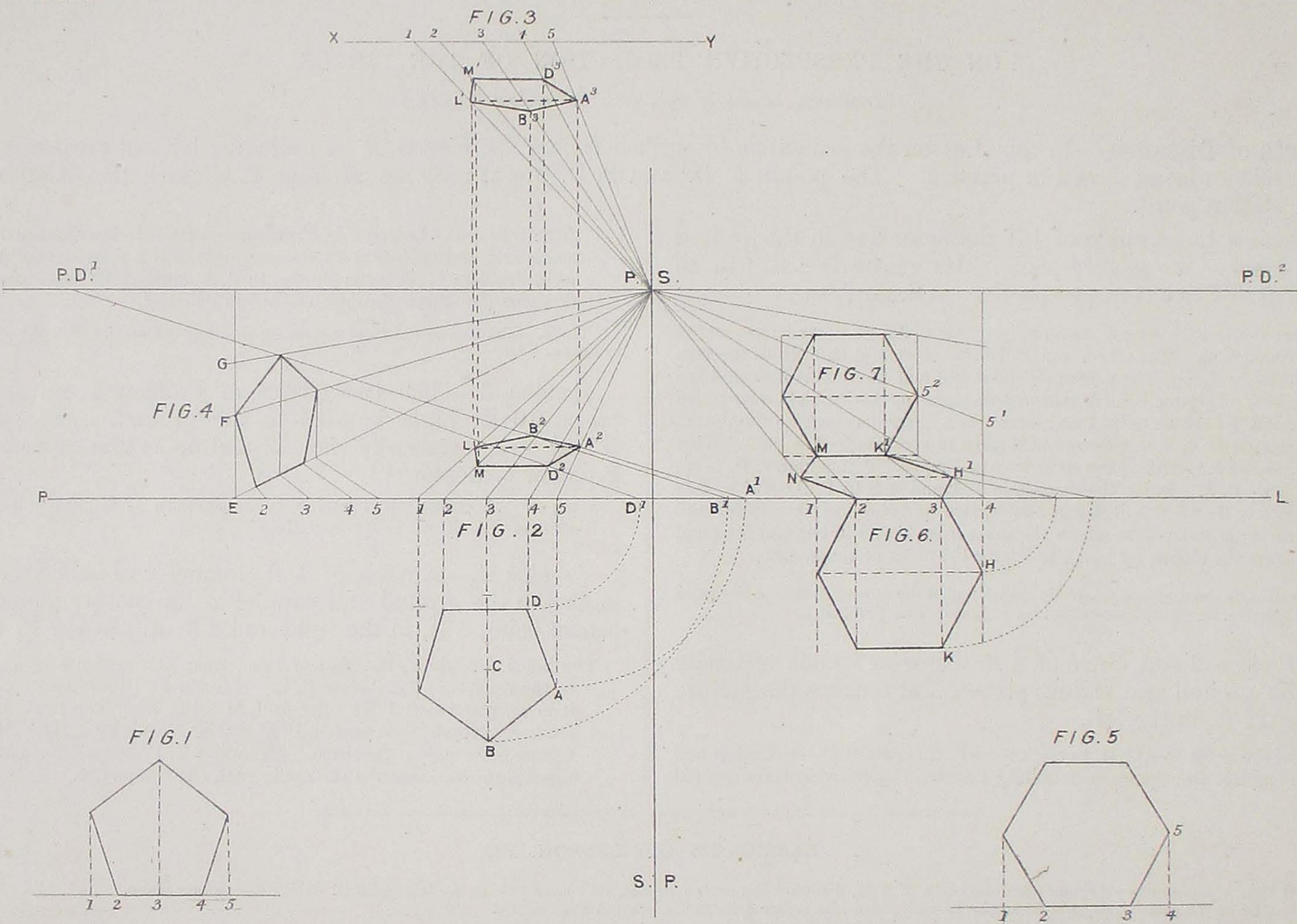
- (1) Give a perspective representation of a circle of 6 ft., diameter lies on the ground, its centre 4 ft. to the left and 3 ft. within the picture plane.
- (2) Also of the same circle at right angles to both planes, its centre 4 ft. to the left and 3 ft. within.
- (3) A similar circle to 1, vertically over it and parallel to the ground plane. Show it in perspective.
- (4) Also of a circle of 5 ft. diameter, perpendicular to the ground and parallel to picture plane, its centre 6 ft. to the right and 3 ft. within the picture plane.

NOTE.—The perspective projection of a circle is only a circle when in a plane parallel to the picture plane. In every other position it appears as an ellipse. (Problem 4.)

SCALE 48

LESSON VIII.

PLATE VIII.



LESSON VIII.

ON THE PERSPECTIVE PROJECTION OF POLYGONS.—(*Height of eye, distance of S. P., &c., as in previous Lessons.*)

Problem 1. (^{*Figs.*}_{*1 & 2*})—A pentagon of 2 ft. sides lies on the ground with one side parallel to the picture plane, its centre being 4 ft. to the left and 4 ft. within the picture plane.

Construct a pentagon of 2 ft. sides as Fig. 1, and let fall the perpendiculars on to a side produced, as 1 2 3 4 5. Set off these points on the P. L., so that point 3 shall be 4 ft. to the left, and join them to P. S. Let fall the dotted perpendiculars from the points 1 2 3 4 5 in P. L., and place the PLAN of the pentagon as shown, so that the centre C fulfils the conditions of the Problem (4 ft. within). It is already 4 ft. to the left. From centre 4 and distance 4 D describe arc D D¹. Join D¹ to P D 1, cutting the line 4, P S in D². Similarly from points 3 and 5 obtain points A², B². Now complete, as shown.

Problem 2. (^{*Fig.*}_{*3*})—A pentagon equal to the last is parallel to the ground and vertically over it 11 ft. high.

The points 1 2 3 4 5 on P. L. are set up vertically, 11 ft., and joined to P. S., vertical lines from the points D², A², B², cutting the corresponding lines to P. S. in points D³, A³, B³, etc.

Problem 3. (^{*Fig.*}_{*4*})—The same pentagon stands perpendicular to the ground and picture planes 10 ft. to the left, and one angle touching the P. P.

Set off on P. L. the point E, 10 ft. to the left, and join E to P. S., from E mark the Points 2 3 4 5 in P. L. equal the same points in Fig. 1, and cut the line from E to P. S. by drawing lines from these points to P. D.¹. Set up the perpendicular E F G, making E G equal to the height of the pentagon in Fig. 1, &c.; join F and G to P. S., and complete as shown.

Problem 4. (^{*Figs.*}_{*5 & 6*})—A hexagon of 2 ft. side has its nearest angle 4 ft. to the right and lies on the ground, with one side touching the P. P.

Construct a hexagon of 2 ft. sides (Fig. 5) and obtain the points 1 2 3 4. Set off on P. L. the points 1 2 3 4, so that point 1 is 4 ft. to the right. Draw the plan so that 2 3 on P. L. is one side. Join the points 1 2 3 4 to P. S. Place the points H and K in perspective, and complete the hexagon 2, 3, H¹, K¹, M, N, as shown.

Problem 5. (^{*Fig.*}_{*7*})—The hexagon on the side M K is the same hexagon standing perpendicular to the ground and parallel to the P. P., and the distance 3 K, within the picture.

The height is set up from point 4 to 5¹, &c., to get point 5², and the hexagon is completed as shown.

NOTE.—This last Figure is a complete hexagon in the same way as the square and circle of Lesson VI. and VII. is a square and circle respectively.

EXERCISES ON LESSON VIII.

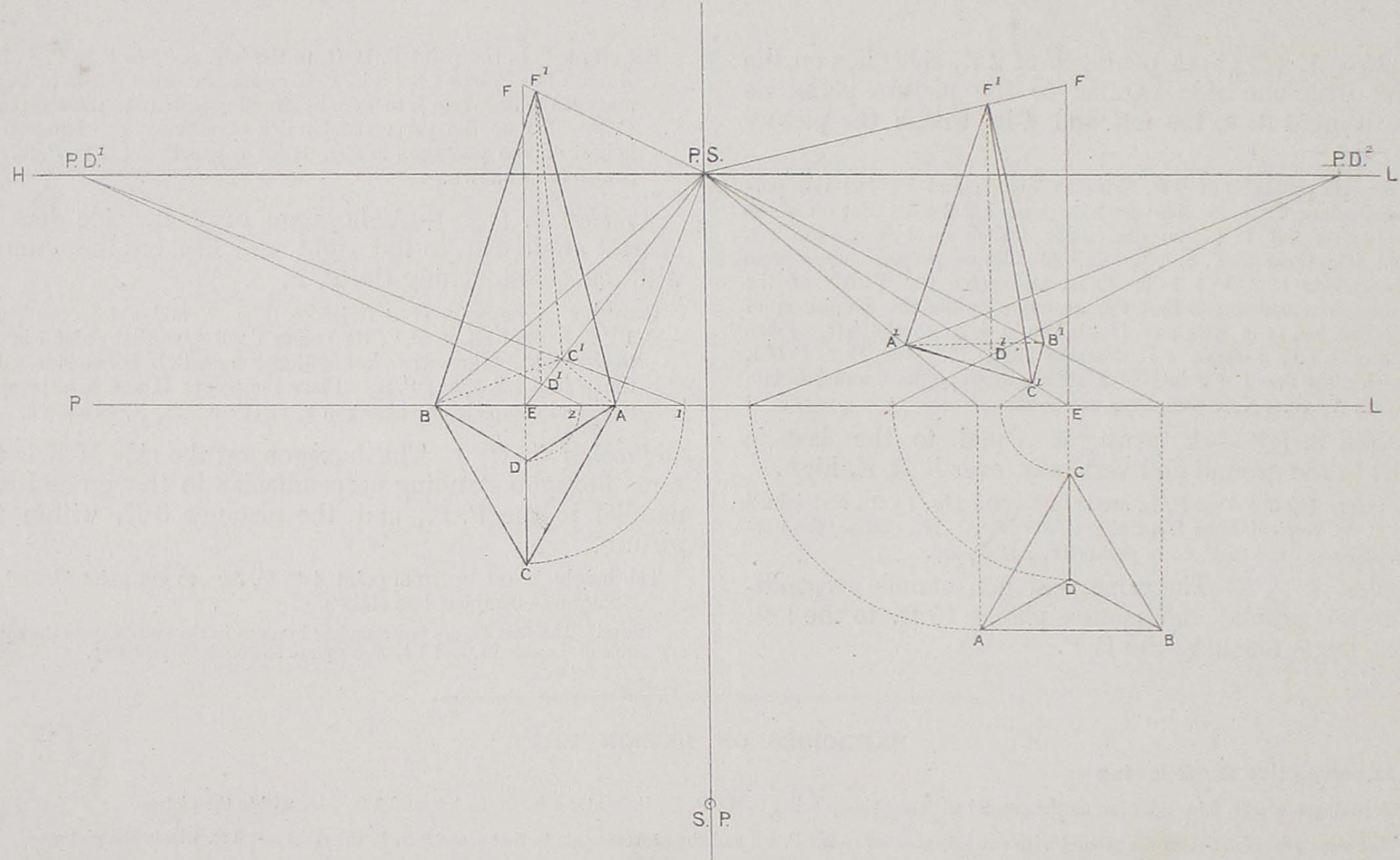
Place in perspective the following—

- (1) A pentagon of 2 ft. 6 in. sides lies on the ground with one side parallel to the P. P., its centre 4 ft. to the left and 3 ft. 6 in. within the picture.
- (2) The same pentagon lies on the ground with one side parallel to the P. P., and the nearest angle to that plane 5 ft. to the right and 2 ft. within the picture.
- (3) A hexagon of 2 ft. 6 in. sides at right angles to both planes, and one angle touching the picture plane 8 ft. to the left.
- (4) An octagon of 2 ft. side lies on the ground, and one side touching the picture plane; its centre to be 5 ft. to the right.

LESSON IX.

PLATE IX.

SCALE $\frac{1}{48}$



LESSON IX.

ON THE PERSPECTIVE PROJECTION OF THE TRIANGULAR PYRAMID.—(*Distance, height of eye, and scale as before.*)

Problem 1. (on the left.)—An equilateral triangular pyramid of 4 ft. sides and 7 ft. high stands on its base, and perpendicular to the ground; one side of the base touches the picture plane with its nearest corner, 2 ft. to the spectator's left.

Draw its plan so that side A B touches the P. L., the point A being 2 ft. to the left of the spectator, as shown in Plate 9. A B C is the plan and point D shows the apex directly over the centre of its base. Place this plan in perspective. It is the triangle A B C¹ and its centre D¹. Now the apex is directly over the centre of its base (D¹). From point E on the picture line set up E F, a perpendicular line 7 ft. high; join F to P. S., and draw D¹ F¹ perpendicular to cut the line F, P. S. in F¹. Then F¹ is the height (7 ft.) in perspective as explained in Lesson V. Problem 2.

Join F¹ to each corner of the base, as F¹ B, F¹ A, and F¹ C¹, this latter line not being seen is shown dotted.

NOTE.—The height is set up from E, because a line from P. S. passes through point D¹ to the picture line, cutting it in E.

Problem 2. (on the right)—The same pyramid to be placed in perspective when the side farthest from the picture plane is parallel, and the nearest angle 8 ft. to the right and 1 ft. 6 in. within the picture.

Place the plan to fulfil the required conditions, viz. :—Point C 8 ft. to the right and 1 ft. 6 in. below P. L., and A B parallel to P. L. Place each point in perspective as before, join the points and the triangle A¹ B¹ C¹ is obtained with its centre D¹. Set up the height E F (7 ft.) from E, because the line from P. S. passes through D¹ to cut the picture line in E. Join F to P. S., and complete as in Problem 1 of this Lesson. *NOTE.*—In this case two faces are seen and all three edges.

EXERCISES ON LESSON IX.

An equilateral triangular pyramid of 5 ft. sides and 8 ft. high is to be placed in perspective under the following conditions :—

- (1) So that one side is parallel to the P. P. and 1 ft. 6. in. within the picture; its nearest corner 4 ft. to the right.
- (2) So that its nearest angle is 2 ft. within and 7 ft. to the left, and its furthest side parallel to the P. P.

A square base pyramid of 4 ft. sides and 10 ft. high is to be placed in perspective :—

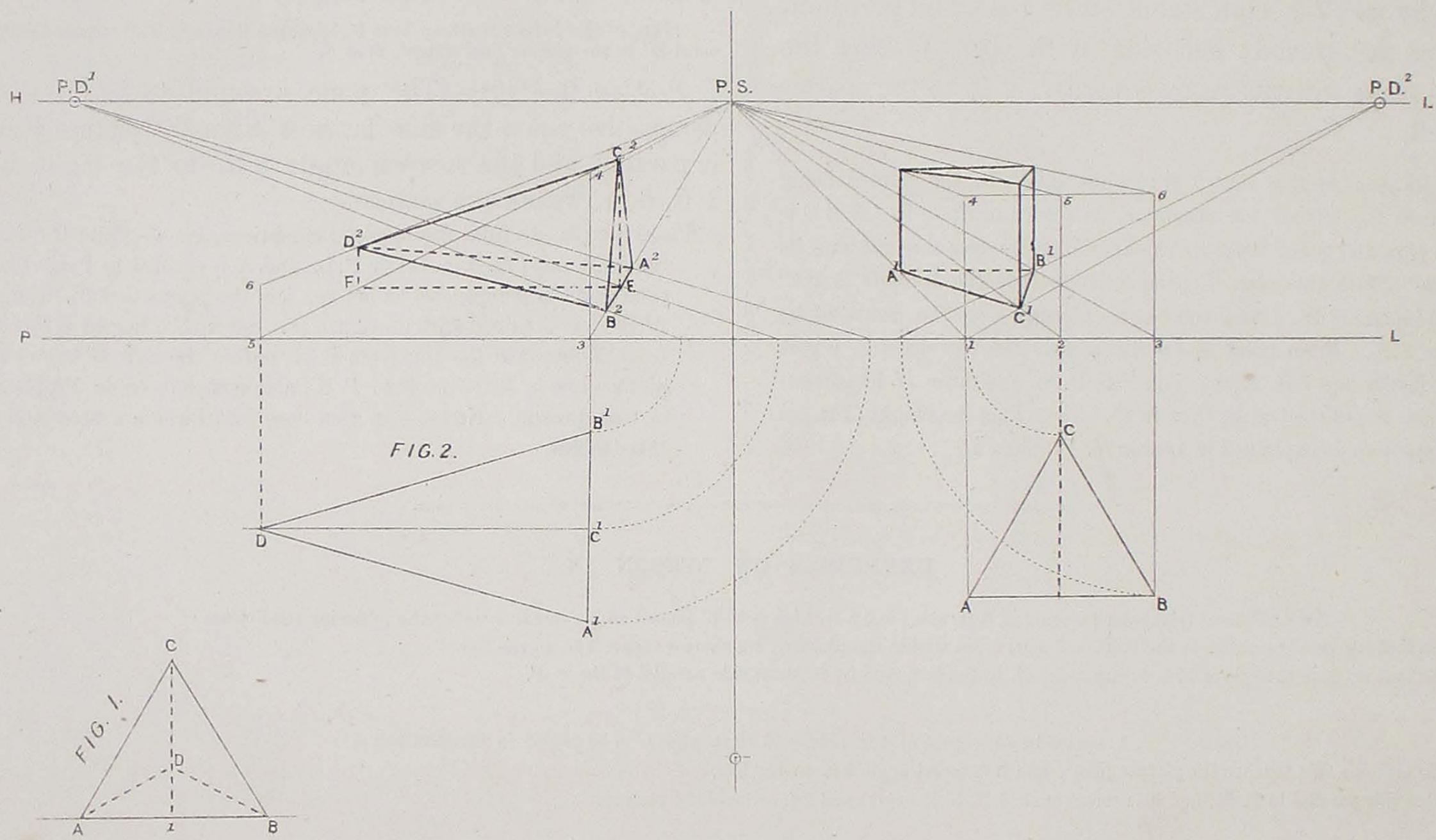
- (1) So that one side touches the picture plane, and its nearest angle 3 ft. to the left.
- (2) One side parallel to P. P., and its nearest angle 4 ft. to the right and 2 ft. within the picture.

Practice with other pyramids, such as pentagonal, hexagonal, &c.

LESSON X.

PLATE X.

SCALE $\frac{1}{48}$



LESSON X.

ON THE PERSPECTIVE PROJECTION OF THE TRIANGULAR PYRAMID, AND TRIANGULAR PRISM.

(*Scale, &c., as before.*)

Problem 1.—An equilateral triangular pyramid is turned over until its base is at right angles to the ground and picture planes. Place it in perspective when the nearest corner of its base is 3 ft. to the left, and 2 ft. within the picture. Each side of its base is 4 ft. long, and it is 7 ft. high.

Commence by drawing the equilateral triangle A B C, and find its centre D (Fig. 1). It will be seen, from this figure, that when A B C is at right angles to the ground, the apex D will be the height D 1 above the ground.

Place the plan, Fig. 2, so that A¹ B¹ is perpendicular to P. L., and point B¹ 3 ft. to the left and 2 ft. below P.L. Put A¹ B¹ in perspective, (A² B²) set up the height 3.4=1 C of Fig. 1, and complete the

end A² B² C² as shown. Set off 3.5=7 ft., and the perpendicular 5.6=1 D (Fig. 1). The point D² is obtained by drawing the line E F parallel to the P. L., and F D² a vertical line as shown. Complete the pyramid by drawing the lines D² C², D² B², and D² A², this last, a dotted line, not being seen.

Problem 2.—An equilateral triangular prism of 4 ft. sides and 3 ft. high is here shown, when the angle C is 7 ft. to the right and 2 ft. within the picture, and the side A B parallel to the P. P.

The plan A B C is first placed in the position required in the question, and then put in perspective A¹ B¹ C¹ (Lessons III. and IV.), the height 3 ft. is then set up on P. L. as 1.4, 2.5, 3.6, and joined to P. S., the perpendiculars being drawn up from A¹ B¹ C¹ to cut the lines to P.S. The prism is completed by joining the points as shown.

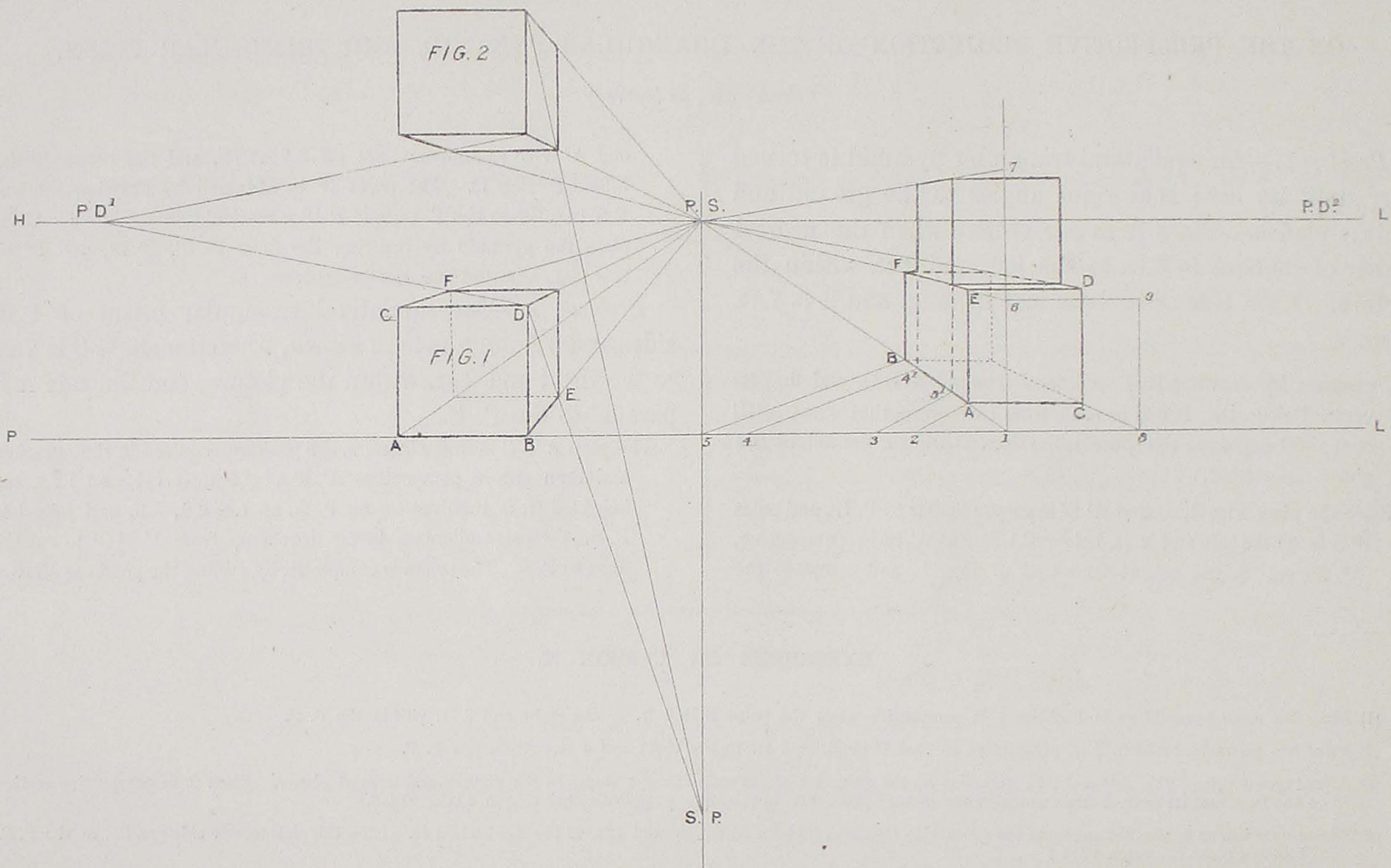
EXERCISES ON LESSON X.

- (1) Place the same pyramid as in Problem 1 in perspective when the point B is 4 ft. to the right and 3 ft. within the P. P.
- (2) Place the prism in Problem 2 in perspective so that C shall be 5 ft. to the right and 4 ft. within the P. P.
- (3) A pentagonal prism, 2 ft. side and 7 ft. long, lies on the ground with its ends at right angles to the picture and ground planes. Place it in perspective resting on one face, and its nearest angle touching the picture plane 4 ft. to the left. (Application of Lesson VIII., Fig. 3.)
- (4) The same prism as 3, at right angles to the ground, so that its axis touches the ground 4 ft. to the right and 4 ft. within the picture, one side parallel to the P. P. (Problems 1 and 2, Lesson VIII.)

SCALE 48

LESSON XI.

PLATE XI.



LESSON XI.

ON THE PERSPECTIVE PROJECTION OF THE CUBE AND RECTANGULAR SOLIDS.—(*Scale, &c., as before.*)

Figure 1 (on the left) shows a cube, 3 ft. edge, on the ground, one face touching the P. P., and its nearest angle B 4 ft. to the left. The face A B C D, touching the P. P., is a square of 4 ft. side. The corners E and F are obtained by drawing the diagonals from D to P. D¹. for F, and S. P. for E (Lesson VI., Problems 1 and 2).

Figure 2 is the same cube raised 7 ft. above the ground. The construction is similar to Fig. 1, as shown.

On the right.—A cube of 3 ft. edge rests on a rectangular block whose end is 3 ft. square, and the block is 5 ft. long, their centres coincide. Place both solids in perspective when the end of the lower block is 7 ft. to the right, parallel to the P. P., and 2 ft. within the picture.

NOTICE in this figure, that the plan is not drawn below P. L. in position, as in previous Lessons. The student will have noticed that, by setting the distances off on P. L., is the same as turning the points on to P. L. after the plan has been drawn below the P. L.; for instance—the rectangular block is 7 ft. to the right and 2 ft. within the picture. Set off 7 ft. to the right in the point 1, and join to P. S., and set back the distance $1.2=2$ ft., join to P. D.² to cut the former line in A; A is 7 ft. to right and 2 ft. within. Mark off $2.5=5$ ft. the length of the block, and cut the same line in B. A B is 5 ft. long in perspective. Set off $3.4=3$ ft. the side of the cube, so that the distance 3.2 and 4.5 shall be the same, that is, so that the centre of the side of the cube shall be over the centre of the side A B as required. Join to P. D.², cutting A B in $4^1 5^1$. At the point 1, set up the heights 1.6 and 6.7, each 3 ft., and join to P. S., and from A B and $5^1 4^1$ draw the perpendicular lines, as shown, to meet the lines from 6 and 7 to P. S. Complete the solids by setting off 1.8=3 ft., join to P. S. and draw A C parallel to P. L. to cut it in C, etc.

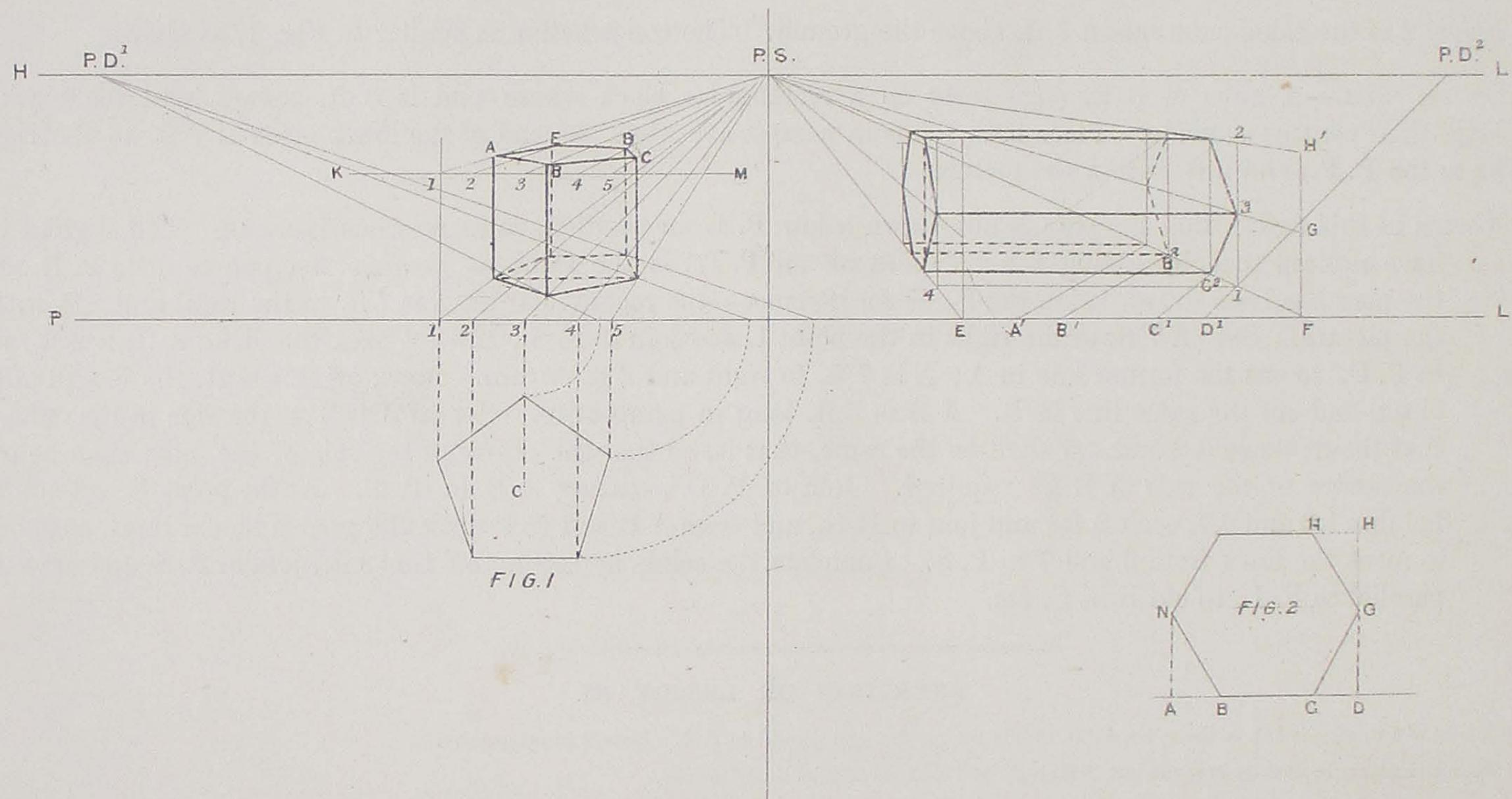
EXERCISES ON LESSON XI.

- (1) A cube of 4 ft. edge is 4 ft. to the left and 3 ft. within the picture, and parallel to P. P. Place it in perspective.
- (2) Place a similar cube directly over the last 10 ft. high, and one face parallel to the ground.
- (3) A rectangular block, 7 ft. long and 4 ft. square (end), is to be put in perspective, when the longer face is parallel to the P. P., and nearest corner 5 ft. to the right and 3 ft. within the picture.
- (4) On the last figure place a cube of 4 ft. edge, directly over its centre, with a side parallel to the P. P.

SCALE $\frac{1}{48}$

LESSON XII.

PLATE XII



LESSON XII.

ON THE PERSPECTIVE PROJECTION OF THE PENTAGONAL AND HEXAGONAL PRISM.

Figure 1 (on the left.)—This is the plan of a pentagonal prism of 2 ft. side and 3 ft. high, having its centre C, 5 ft. to the left, and 3 ft. 6 in. within the picture, and one side parallel to the picture plane.

The vertical dotted lines are drawn from each angle to the P. L. in the points 1 2 3 4 5, and the plan is put in perspective on the ground. (Lesson VIII., Problem 1).

The upper face is obtained as in Problem 2, Lesson VIII., thus : Set up the height from point 1 on P. L. = 3 ft., the height of the prism. Draw the line K M parallel to P. L. through point 1, and mark the points 2 3 4 and 5 on it. Join these points to P. S. Draw the vertical lines from each angle on the ground to meet their respective lines, giving the points A B C D E. Darken as shown.

Figure 2 (on the right.)—An hexagonal prism of 2 ft. side and 7 ft. long rests on one face on the ground. Its edges being parallel to P. P., and its nearest angle being 4 ft. to the right and 2 ft. within.

Figure 2 shows the end of the hexagonal prism of 2 ft. side. One side B C is produced, and the vertical lines N A and G D let fall on it.

Set off the point E on P. L. 4 ft. to the right, and E F 7 ft. long. Join E and F to P. S. Set back F D¹ = 2 ft. (distance within P. P.) Join D¹ to P D², cutting the former line in 1. Draw the horizontal line 1, 4. From D¹ in P. L. set off D¹ C¹, C¹ B¹, B¹ A¹ = D C, C B, B A (Fig. 2). Join to P D² to cut the line from F to P. S., in C², etc. Set up F H¹ = twice D G (Fig. 2) and join to P. S. Draw the vertical lines from C², B², etc., and complete the end of the hexagonal prism as shown. By drawing the horizontal lines from the points B², C², etc., to meet the line from E to P. S. the other end is obtained. Complete, by darkening the visible edges, as shown.

NOTE.—The points D¹ C¹ B¹ A¹ could be set off from E if the student pleases. It is not done so here, in order to keep those points clear of the Figure on the left.

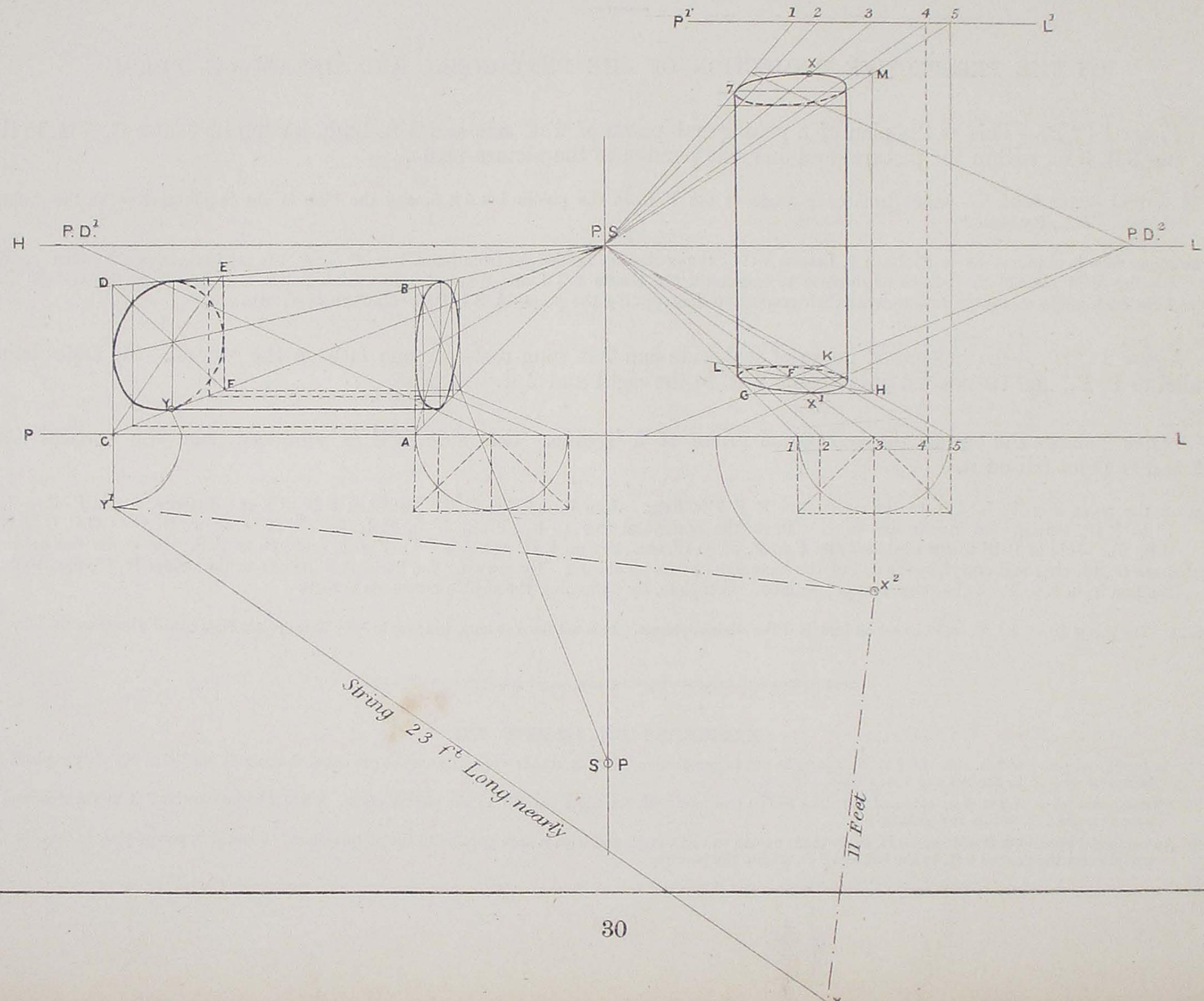
EXERCISES ON LESSON XII.

- (1) An hexagonal prism of 3 ft. side and 3 ft. high is to be put in perspective when it stands vertically on the ground with one side parallel to the picture plane, the centre being 5 ft. to the left and 4 ft. within.
- (2) A pentagonal prism of 2 ft. 6 in. side and 7 ft. long lies on one face with its edges parallel to the picture plane. Place it in perspective 3 ft. to the right and its nearest angle 2 ft. within the picture.
- (3) An octagonal prism of 2 ft. side and 8 ft. long stands on one end on the ground with a face parallel to the picture plane. Place it in perspective, so that the axis would enter the ground 6 ft. to the left and 5 ft. within the picture.

LESSON XIII

PLATE XIII

SCALE $\frac{1}{8}$



LESSON XIII.

ON THE CYLINDER.

Problem 1 (on the left).—A cylinder, 4 ft. diameter and 8 ft. long, lies on the ground, with its axis parallel to the picture plane. The cylinder touches the picture plane, with its nearest end 5 ft. to the left.

Set off A 5 ft. to left, and join to P. S. Describe the semicircle with radius 2 ft., so that it may touch at A, as shown. Surround it with rectangle, as explained in Lesson VII., and join each point from the P. L. to P. D.¹. Set up the height A B = to the diameter (4 ft.) Draw the diagonals and describe the oval as in Lesson VII. Set off A C = 8 ft., and C D 4 ft. Complete the square C D E F, draw the diagonals, and complete the ellipse as before.

NOTE.—In Lesson VII., Problem 2, the semicircle is described on the perpendicular line C D, and here below P L, from the point A. Both are correct. Sometimes it is more convenient to place it in one position and sometimes in another, to avoid getting a number of lines crossing the figures, which is confusing to the young student.

Problem 2 (on the right).—A cylinder, 4 ft. diameter and 11 ft. long, stands on its end on the ground, with its axis parallel to P. P., or which is the same thing as perpendicular to the ground. Its axis enters the ground at a point 7 ft. to the right, and 6 ft. within the picture.

Find the point F 7 ft. to the right and 6 ft. within the picture (Lesson III). Describe the semicircle, with radius 2 ft. from point 3, which is 7 ft. to the right; surround it with the rectangle, and join each point to P. S. The line passing through the centre F to P. D.² is one diagonal of the square. Where it cuts the lines from points 1 and 5 to P. S., in G and K, draw the horizontal line G H and K L, which completes the square. The diagonal H L is then drawn, and the ellipse described by hand through the points, where the diagonals cut the lines from points 2 and 4. Set up the height 5.5 = 11 ft., and mark the points 1 2 3 4 5 on the horizontal line P.¹ L.¹ Join to P. S., and draw H M, a perpendicular line meeting the line from 5 to P. S. Complete the square as before, and describe the ellipse. Join the sides as shown.

Problem 3.—What length of line will reach from point Y on the ground to point X at the top of the cylinder, on the right?

First find the point X on the ground. It is X¹. Now take X¹ and Y out of perspective (Lessons III. and IV.) They are X² Y¹, about 20 ft. apart. Draw X² X perpendicular to Y¹ X², making X² X = 11 feet. The length of the string is the hypotenuse of this right angled triangle, viz. : X Y¹, which, measured to the scale $\frac{1}{8}$, is nearly 23 ft. long.

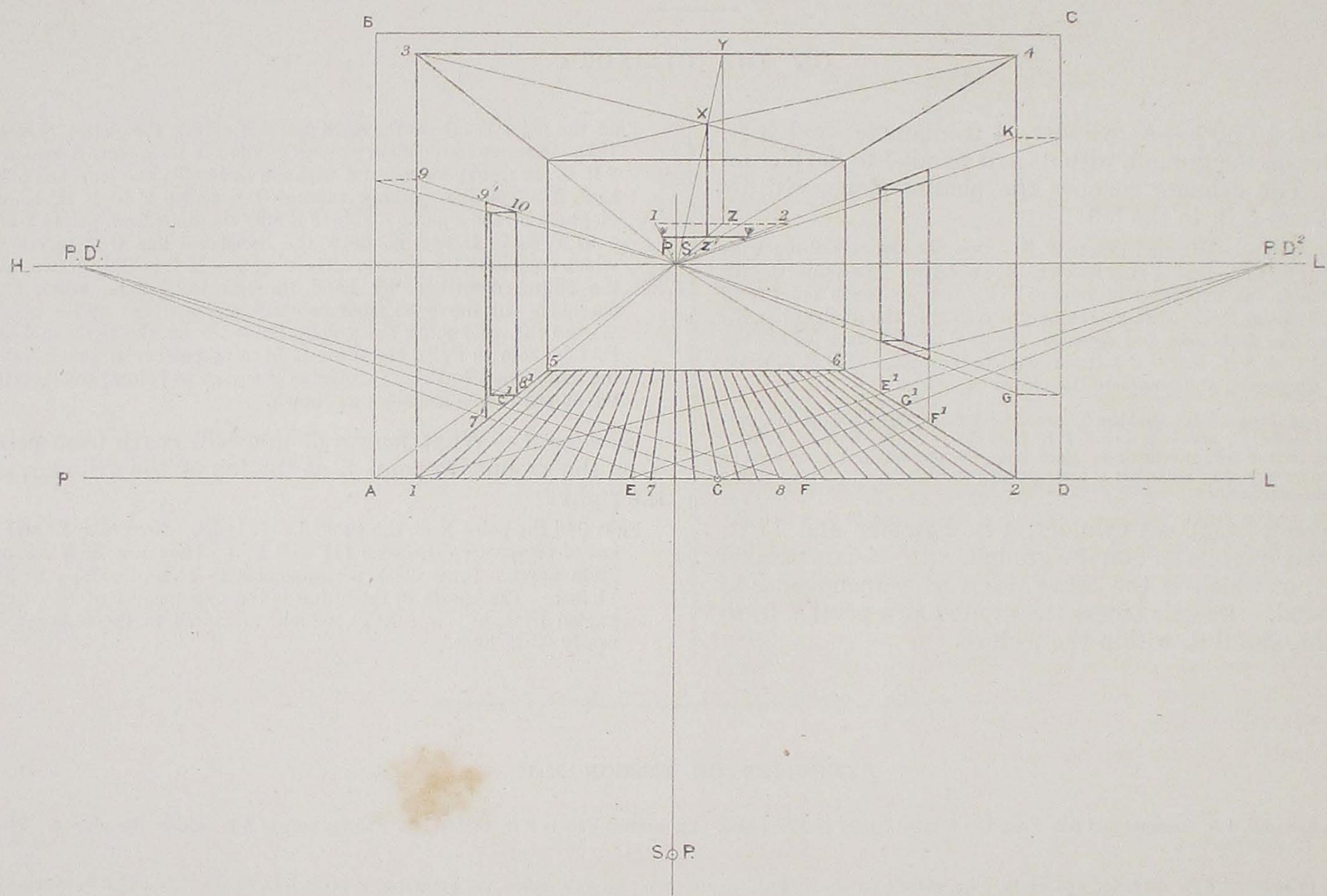
EXERCISES ON LESSON XIII.

- (1) A cylinder, 4 ft. diameter and 6 ft. long, lies parallel to the picture plane. Its nearest end is 3 ft. to the left, and its axis is 4 ft. within the picture. Place it in perspective.
- (2) A cylinder of 4 ft. diameter and 10 ft. long, stands on the ground, on its end. Its axis enters the ground at a point 6 ft. to the right and 5 ft. within the Picture. Place it in perspective.
- (3) What length of line will reach from a point 7 ft. to the left and 1 ft. within the picture to the point corresponding to X in Problem 3 of this Lesson (Exercise 2).

SCALE $\frac{1}{48}$

LESSON XIV.

PLATE XIV



LESSON XIV.

ON THE PERSPECTIVE PROJECTION OF A RECTANGULAR ROOM.

(*Scale, &c., as before.*)

A room 14 ft. square and 10 ft. high is here shown in perspective. The spectator stands 6 ft. from the left hand wall. Draw the rectangle 1 2 3 4, so that point 1 is 6 ft. to the left, and the distance 1 2 = 14 ft. long; also 1 3 and 2 4, 10 ft. high. Join each point to P.S., and draw the diagonal from 1 to P.D.² cutting the line from 2 to P. S. in 6. 5.6 is the width of the back wall, which complete as shown.

The flooring boards are each 6 inches wide. Set them off on 1.2, and join each to P. S. The board from point C is the centre board of the room, and as the room is square, the line from C to P. D.¹ and P. D.² gives the centre of the walls on the right and left (C¹ C²). In the centre of the left hand wall is a door, 3 ft. wide and 7 ft. high. Set off 1 ft. 6 in. on each side of C in P. L. in the points 7 and 8, and join to P.D.¹, cutting the line 1.5 in 7¹, 8¹ the width of the doorway. Make the height 1 9 = 7 ft. and join to P.S., cutting the vertical lines from 7¹ and 8¹ in 9¹, 10, then 7¹ 8¹ 9¹ 10 is the doorway. To show its thickness, suppose the wall to be 1 ft. thick; make the distance I A = 1 ft., then the line from A to P. S. will determine the thickness from 8¹ and 10, as shown. In the right wall is a window, 4 ft. wide and 6 ft. high, with its lower part 2 ft. above the floor. Set off

C E and C F, each 2 ft., and join to P. D.². It gives points E¹ F¹. Set up 2 G = 2 ft. and G K 6 ft., and join to P. S. Complete the window as shown.

A 2-light pendant, 4 ft. long, hangs down from the centre of the ceiling, each of its arms being 1 ft. 6 in. long. From points 3 and 4 draw the diagonals of the ceiling, cutting in X. From P. S. draw through X the line X Y. Let fall the perpendicular Y Z = 4 ft. Join Z to P. S. Then X Z¹ is the length of the chandelier in perspective. Set off Z 1 and Z 2 each 1 ft. 6 in. for the arms of the chandelier, and join to P. S., and draw the arms in perspective through Z¹, as shown.

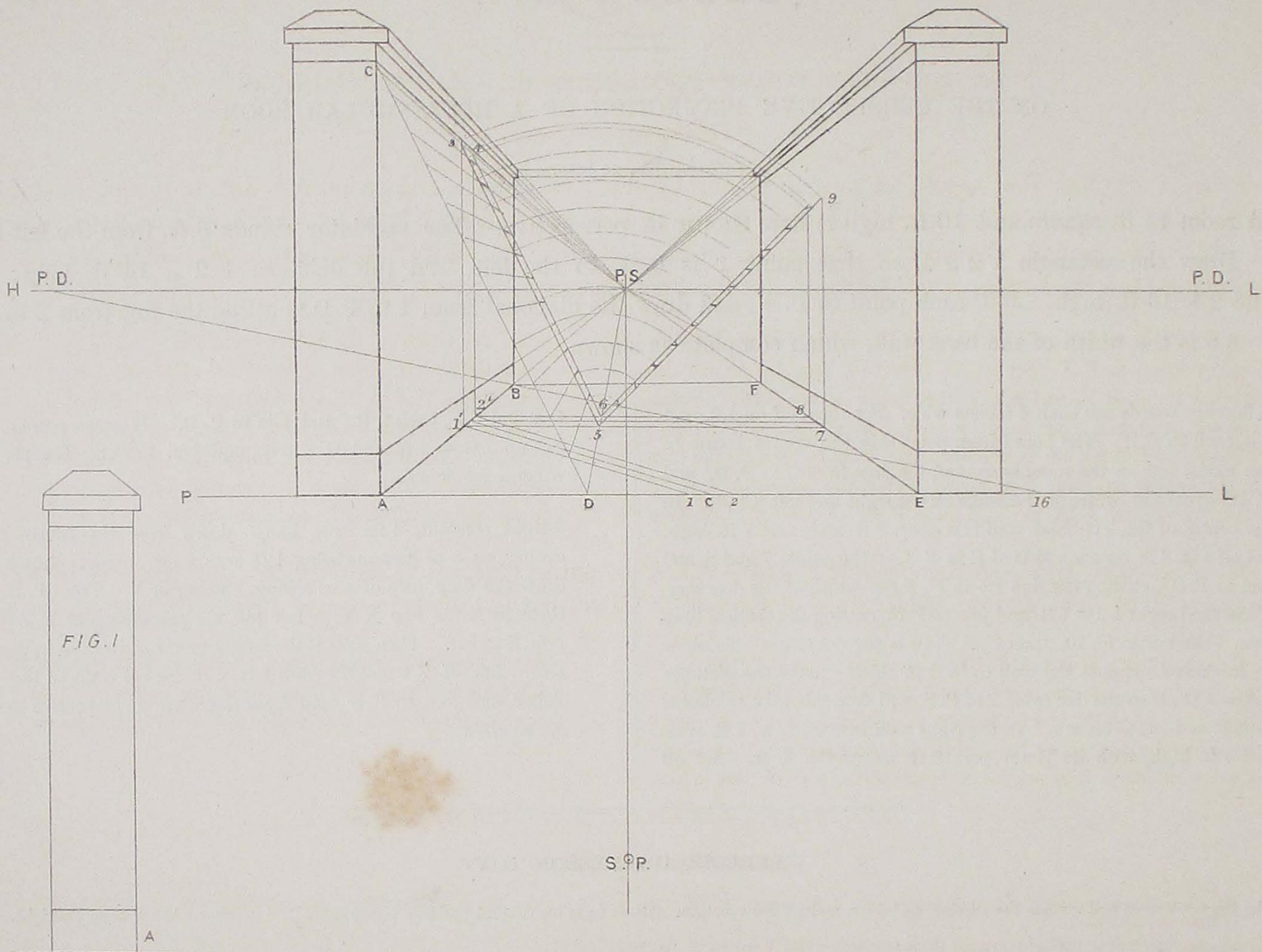
EXERCISE ON LESSON XIV.

Take the same room and reverse the window and door, and let the spectator remove from his present position to a point 2 ft. 6 in. nearer the wall on his right.

SCALE $\frac{1}{48}$

LESSON XV

PLATE XV



LESSON XV.

ON THE PERSPECTIVE PROJECTION OF LEANING LADDERS.

(*Scale, &c., as before.*)

Figure 1 (*on the left*) is the section of a wall as cut by the picture plane; the point A is 6 ft. to the left, and the wall is 16 ft. long. Against it, and midway between its two ends rests a ladder, 1 ft. wide, touching the wall at a point 10 ft. 6 in. above the ground, the foot of the ladder being 5 ft. from the wall. The bars of the ladder are 1 ft. apart, commencing from the bottom of the ladder. Put the whole in perspective.

Draw Figure 1 on the picture line, so that A is 6 ft. to the left. Join as shown to P.S. Set off $A - 16 = 16$ ft. long. Join to P. D.¹ It gives the point B, and A B is the length of the wall. Darken the wall as shown. Set up A C = 10 ft. 6 in. and A D = 5 ft. Join C D, and on C D set off feet commencing from D, because the bars of the ladder are each 1 ft. apart. Find the centre of A B by taking the centre A 16, in C. Join to P. D.¹ Set off C 1 C 2, each

6 in., the width of the ladder being 1 ft., and join to P. D. This gives the width of the ladder $1^1 2^1$. Draw the perpendicular line from points $1^1 2^1$ to cut the line from C to P. S. in 3 and 4. Also draw the horizontal lines from $1^1 2^1$ to cut the line from D to P. S. in 5 and 6. Join 3 5 and 4 6. These show the sides of the ladder in perspective. The staves are obtained by lines from the points previously set off on C D, and now joined to P. S. as shown.

The Wall on the right.—A corresponding wall is also to be put in perspective on the right, the roadway between the walls being 13 ft. wide, and the ladder fixed at the foot as above, but the top turned over against the wall on the right.

On P. L. make the width A E, 13 ft., and complete the wall as in the last case. At the points 7 and 8 where the horizontal lines from $1^1 2^1$ cut E F, erect the two perpendicular lines, and since the ladder is fixed at its foot, place the compass point at 5 and describe the arc 3 9, which latter point is the top of the ladder.

Join 9 5 one side of the ladder. Similarly from point 6, the other side may be obtained. Each bar is then carried over by means of arcs described from the centre 5, and the bars drawn to vanish to P. S.

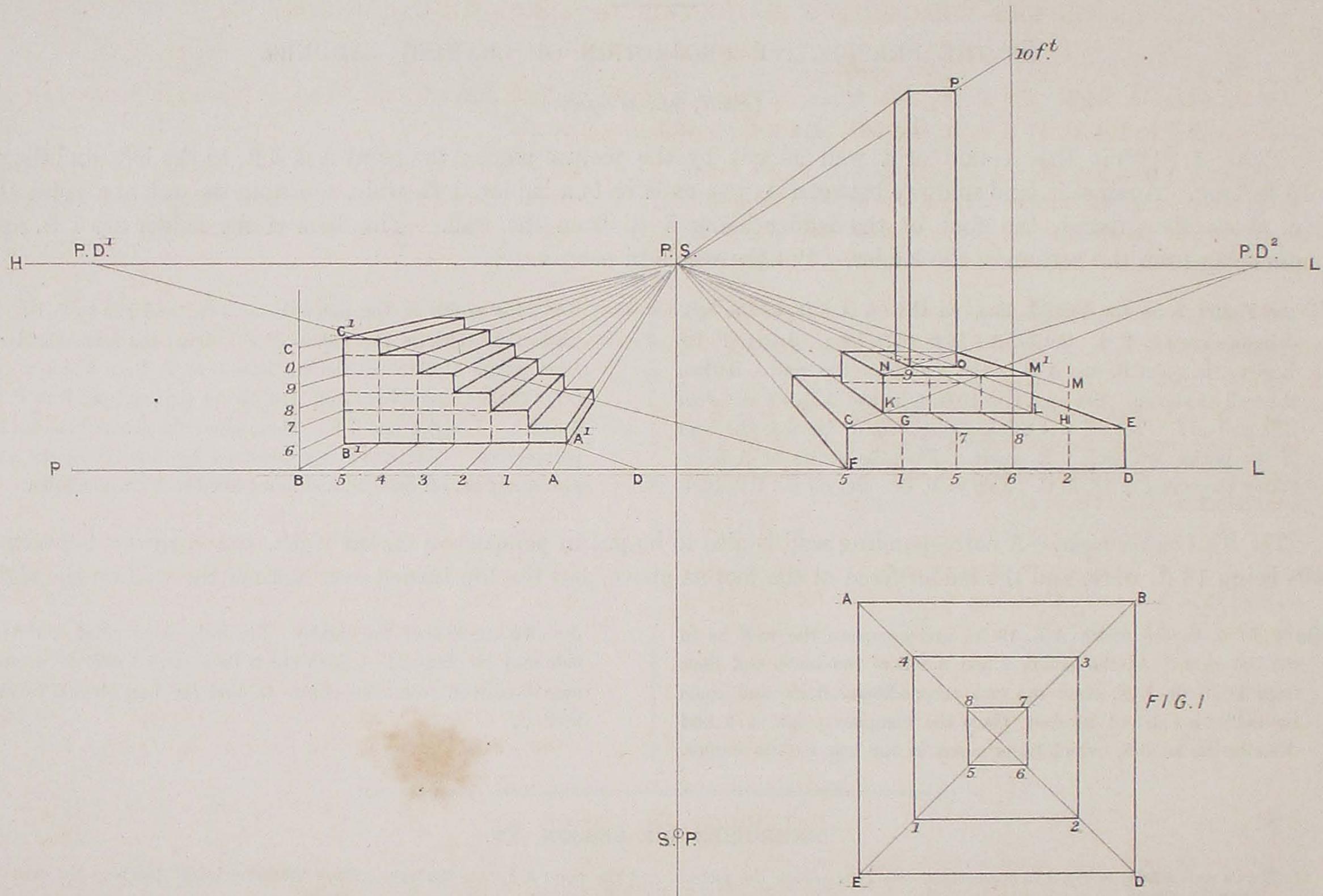
EXERCISES ON LESSON XV.

- (1) Place a wall similar to Fig. 1 in perspective, but 1 ft. within the picture, and the point A 8 ft. to the right. Show the same ladder touching the wall 8 ft. from the ground.
- (2) Suppose the foot of the ladder fixed, show the ladder laid down on the ground.

LESSON XVI.

SCALE $\frac{1}{48}$

PLATE XVI



LESSON XVI.

ON THE PERSPECTIVE PROJECTION OF RECTANGULAR BLOCKS, &c.

(*Scale, etc., as before.*)

On the left.—A flight of 6 steps, 5 ft. wide, having a rise of 6 inches, and a 9-inch tread, is placed in perspective with the end parallel to the P. P. 3 ft. to the left, and 2 ft. within the picture.

Set off point A, 3 ft. to left, join to P. S., cut this line in A¹, 2 ft. within the picture; set off the distance A 1, 1.2, 2.3, etc. = 9 inches—(the tread), and at B, which is the end of the 6th step, set up the vertical line B C, and set off on it six—6 inches by scale—(the

rise); draw the lines A¹ B¹ C¹, and complete the end A¹ B¹ C¹ as shown. Set off D F = 5 ft.—(the width), join to P. D.¹, and complete the other end as shown in the figure.

On the right.—In Fig. 1, A B E D and 1 2 3 4 are square blocks of stone, each 1 ft. thick. On the top block rests the rectangular piece, 5, 6, 7, 8, 8 ft. long; put them in perspective when E D touches the picture plane, and the angle E is 4 ft. to the right.

Set off F D on P. L. so that F is 4 ft. to the right and F D = E D, Fig. 1, and complete the rectangle C E D F, making C F., 1 ft., join to P. S. From C, draw the diagonal to P. D.², and complete the lower block as in the figure.

Set off 1.2 on P. L. between D F, equal to 1.2 in Fig. 1 in the position shown; draw 1 G and 2 H vertical lines to meet the top edge of the lower block in G and H. Join them to P. S.; the line from G cuts the diagonal from C in K; draw the horizontal line, K L, it is the edge of the base of the upper block. Set up H M = 1 ft., and from L draw L M¹, meeting the line from M to P. S.; (notice carefully how these heights are obtained); complete the face

K L M¹ N, and draw the diagonal from N to P. D.², and complete the upper block as shown.

Set off 5.6 between 1.2 on P. L. = 5.6 of Fig. 1; draw the vertical lines 5.7 and 6.8, from 7 and 8, the lines to P. S. meeting K L, then the vertical lines to meet M¹ N, and then vanish to P. S., cutting the diagonal from N in point 9. Complete the small square as shown. Proceed thus for the height of the block 5.6.7.8:—it is 8 ft. high, and stands on two blocks each 1 ft. thick, therefore the top is 10 ft. above the ground;—follow the point O out of perspective on to the P. L., it is point 6. Set up the vertical line from 6, 10 ft. high, and join to P. S. Draw O P, a vertical line meeting it in P, and complete the other edges as shown.

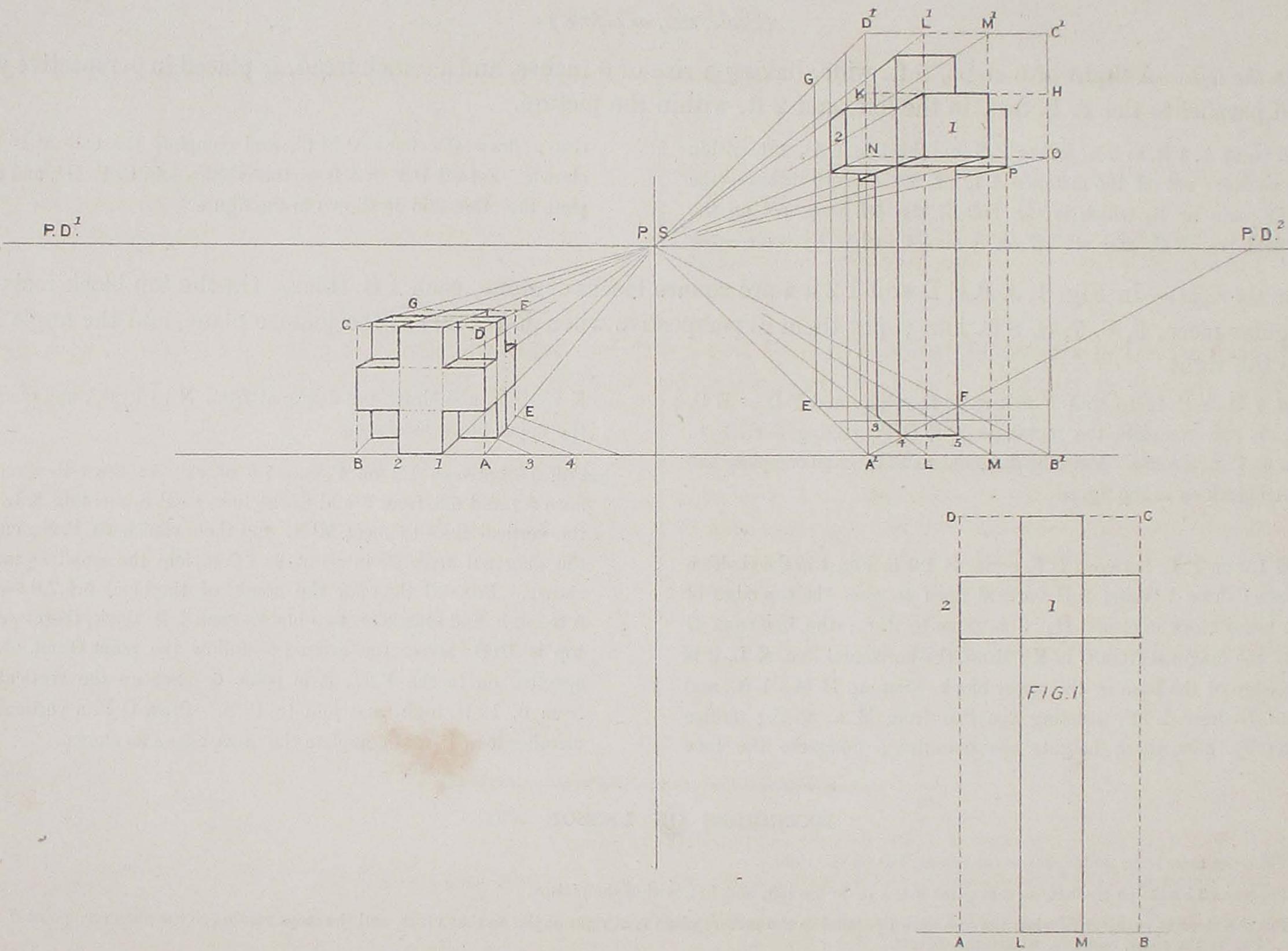
EXERCISES ON LESSON XVI.

- (1) Place the same steps under similar conditions, but on the right.
- (2) Place the 3 blocks on the left, so that point F is 4 ft. to the left, and 1 ft. within the picture.
- (3) Put the steps in perspective when the end shown parallel to the picture plane is at right angles to that plane, and the steps parallel to the observer, point A to be 5 ft. to the right, and 2 ft. within the picture.

SCALE $\frac{1}{48}$

LESSON XVII.

PLATE XVII.



LESSON XVII.

(*Scale, etc., as before*).

On the left.—On each face of a cube of 3 ft. edge is cut a cross, 1 ft. deep, and each arm 1 ft. wide, the face of one cross touches the picture plane 4 ft. to the left.

Place the cube A B C D E F G in perspective so that the face A B C D touches the picture plane, and A is 4 ft. to the left (Lesson XI., Fig 1). On the face A B C D draw the cross, so that the width of the arms (as 1.2) is 1 ft.; set off from A the distances 3.4, each

one foot, and place the cross on the face A D E F. By drawing the lines from the two crosses, as shown in the figure, the cross is obtained on the top face C D F G. Complete as shown in the figure.

On the right.—Fig 1 is the elevation of a solid cross, with four arms, one arm being at right angles to each face of the shaft; place it in perspective, so that the nearest face shall be 5 ft. to the right, and the face marked 1 (Fig. 1) touch the P. P.

Draw the dotted line A B C D around the elevation (Fig. 1), and suppose the cross to be cut out of a solid rectangular block, with square end, A B or C D, and length A D or B C. Place such a block in perspective so that point A is 5 ft. to the right as in A¹ B¹ C¹ D¹ E G; on the face A¹ B¹ C¹ D¹, make the cross L M L¹ M¹ H K N O as in Fig. 1 (shown by dotted lines in perspective), and darken the square marked 1. This is the face

touching the P. P. Draw the diagonal A¹ F on the face A¹ B¹ E F of the block on the ground, cutting the lines from L and M to P. S. in 3.4.5, and darken the lines 3.4 and 4.5, the visible edges of the foot of the shaft. Join K and N to P. S., and draw the vertical lines as shown from 3 and 4 produced to A¹ E, and darken the face, 2; this face is 5 ft. to the right. Complete the cross as shown in the figure.

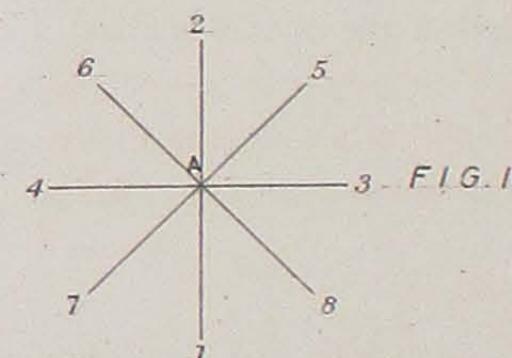
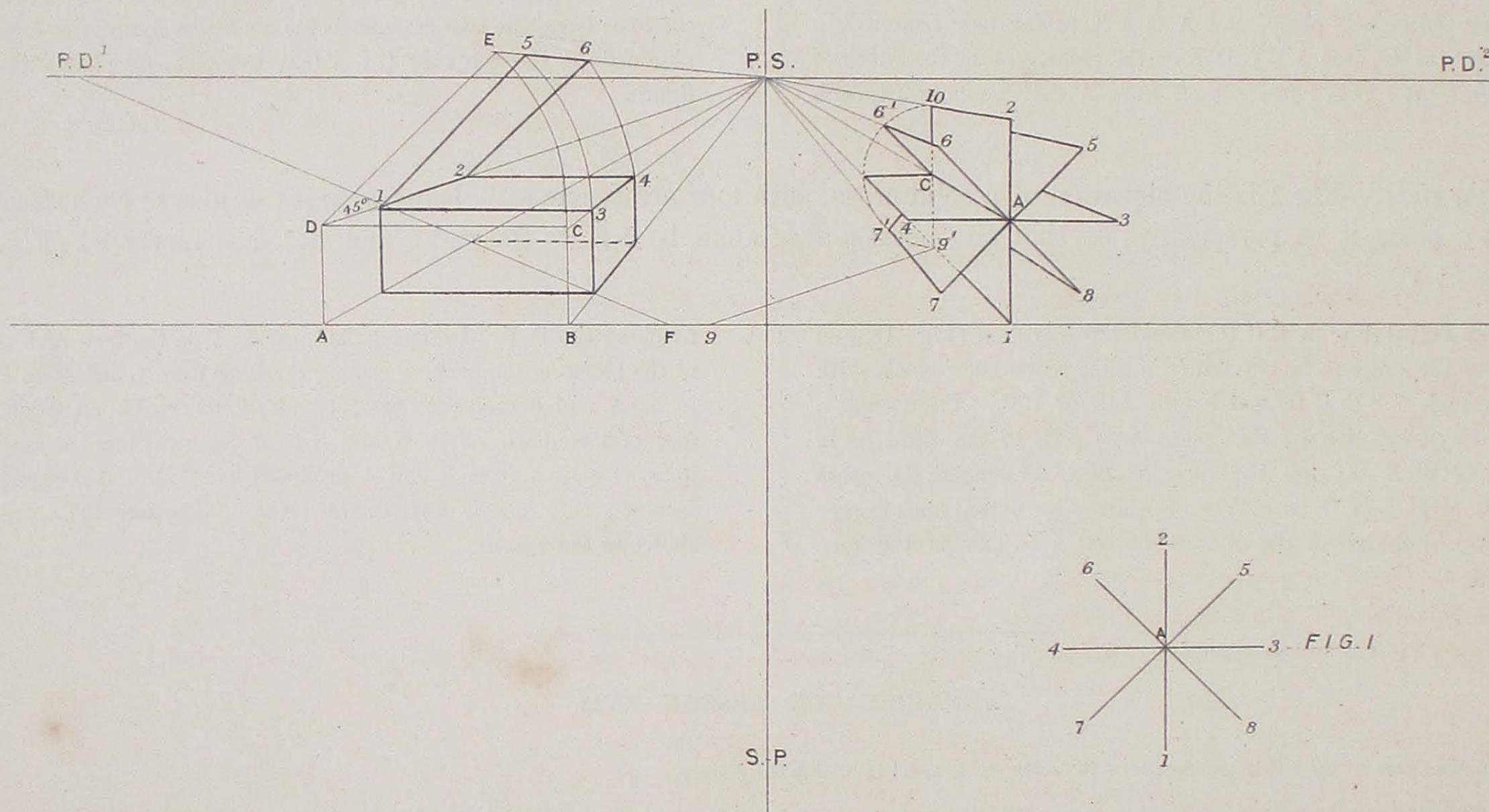
EXERCISES ON LESSON XVII.

- (1) Place the figure on the left in perspective, 4 ft. to the right, and 1 ft. within the picture.
- (2) Place the cross on the right in perspective, so that the point A shall be 5 ft. to the left, and 1 ft. within the picture.
- (3) Place the figure on the left in perspective when the spectator stands opposite the centre of the face A B C D, which face touches the P. P.

LESSON XVIII.
INCLINED PLANES

PLATE XVIII.

SCALE 48



LESSON XVIII.

ON THE PERSPECTIVE PROJECTION OF INCLINED PLANES.—(*Scale, &c., as before.*)

On the left.—A box, 5 ft. square and 2 ft. deep, is parallel to the P. P., the nearest corner being 4 ft. to the left and 2 ft. within the picture. The lid is raised, making an angle of 45° with the ground plane. Place it in perspective.

On the picture plane draw A B C D E the end of the required box (5 ft. by 2 ft.), so that E D C is 45° , and B 4 ft. to the left. Notice the lid D E is the same length as the box, *i.e.*, as D C, hence the use of describing the arc E C from centre D. Set off B F = 2 ft. and complete the perspective projection of the body of the box by drawing lines from A B and C D to P. S., &c.

For the cover:—Draw the line from E to P. S., and from the centre 1 with radius 1 3 cut this line in 5, and from centre 2 with radius 2 4, cut it in 6. 5 6 is the edge of the cover. Complete as shown.

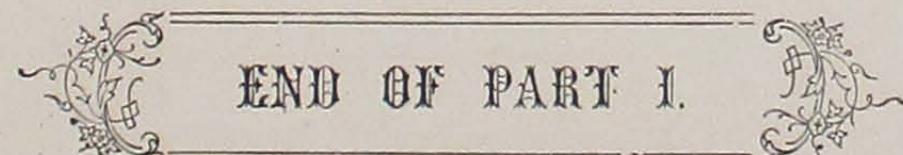
On the right.—Fig. 1 shows the end elevation of eight planes 6 ft. long, intersecting in A. Represent them in perspective when the angle 1 is 5 ft. to the right and the edge 1 2 touches the P. P.

Draw Fig. 1 on P. L. so that the point 1 is 5 ft. to the right. Join from points 1 2 3 4 5 6 7 8 to P. S. Set off 1 9, 6 ft. long; join to P. D.², cutting the line from 1 in 9¹. A vertical line from 9¹ gives

the point 10 and centre C. From the centre C and radius C 10 describe the semi-circle, giving the points 6¹ and 7¹. Complete as shown.

EXERCISES ON LESSON XVIII.

- (1) Place the same box in perspective, showing the cover hinged to the edge 3 4.
- (2) Place Fig. 1 in perspective when point 1 is 6 ft. to the left and 2 ft. within the picture.
- (3) Suppose Fig. 1 to be drawn half size. Place the planes in perspective when point 1 touches the P. P. 6 ft. to the right.



LESSON XIX

ANGULAR PERSPECTIVE

PLATE XIX

SCALE $\frac{1}{48}$

45°

PART II.—ANGULAR PERSPECTIVE.

(45° , the object touching the picture plane.)

LESSON XIX.

Centre of Vision.—In the preceding Lessons one or more sides of the figures have been parallel to the picture plane; they will now be treated when making Angles with that plane, hence we get the term **Angular Perspective**. In this Lesson let the student suppose the spectator standing at the station point (S.P.) as before, 14 ft. from the P. S., or, as more generally called in angular perspective, the **Centre of Vision** (C.V.). Let H.L. be the horizontal and P.L. the picture lines, 5 ft. below it as before, and let the spectator look at an object, such as a *square* of 5 ft. side lying on the ground behind the *picture plane*, one angle touching that plane 3 ft. to his left, and the sides of the square making angles of 45° with it.

Vanishing Points (V.P.)—At the S.P. draw G E parallel to the picture line, and make the angles E P F and G S K each 45° , cutting the horizontal line in V.P. 1 and V. P. 2. These points are called **Vanishing**

Problem 1.—The square of 5 ft. sides, mentioned above, lies behind the picture plane, and touching it at a point 3 ft. to the spectator's left. Show it in perspective; its sides making angles of 45° with the P. P.

Set off on the P. L. the point A 3 ft. to the left, and draw lines from this point to V. P. 1 and V. P. 2. These lines contain the sides of the square.

Set off the distances A B and A C each 5 ft., and from B draw a line

NOTES.—(1) To cut off a distance on a line vanishing to V. P. 1, set that distance on the P.L. and join to M. P. 1, and *vice versa*; because every vanishing point has its own measuring point.

(2) In angular perspective of 45° the V. P. 1, V. P. 2, and S. P. are the same distance from centre of vision; so also must be the distance M. P. 1 and M. P. 2.

(3) The points V. P. 1 and V. P. 2 (45°) occupy the positions of the points of distance in parallel perspective.

(4) One of the diagonals of a square in perspective on the ground (45°) will meet the C. V., and the other be parallel to the picture plane.

(5) All parallel lines vanish to the same point—*e.g.*, the parallel sides of the square.

Points, and the sides of all figures making angles of 45° with the picture plane must be drawn to meet in one or other of these points.

Measuring Points (M.P.)—In parallel perspective, when a line vanishing to the point of sight had to be cut, the distance required was set off on the picture line and drawn back to the point of distance, which point was the **Measuring Point** for horizontal lines in parallel perspective. In angular perspective, each Vanishing Point has its own **Measuring Point**, obtained thus:—from V.P. 1, as centre and distance, to S.P., describe an arc cutting H.L. in M.P. 1—that is, **Measuring Point 1**, and from V.P. 2, as centre and distance, to S.P., describe the arc cutting H.L. in M.P. 2—that is, **Measuring Point 2**. All distances required on lines vanishing to Vanishing Point 1 must be cut off by lines going to Measuring Point 1, and *vice versa*.

to M P 2, cutting that line to V P 2 in B¹: A B¹ is the side of the square in perspective. A line from C to M. P. 1 cuts the line from A to V. P. 1 in C¹. A C¹ is also a side of the square in perspective. From B¹ draw the line to V. P. 1, and from C¹, the line to V. P. 2, cutting in D: then A B¹ D C¹ is the square in perspective.

EXERCISES ON LESSON XIX.

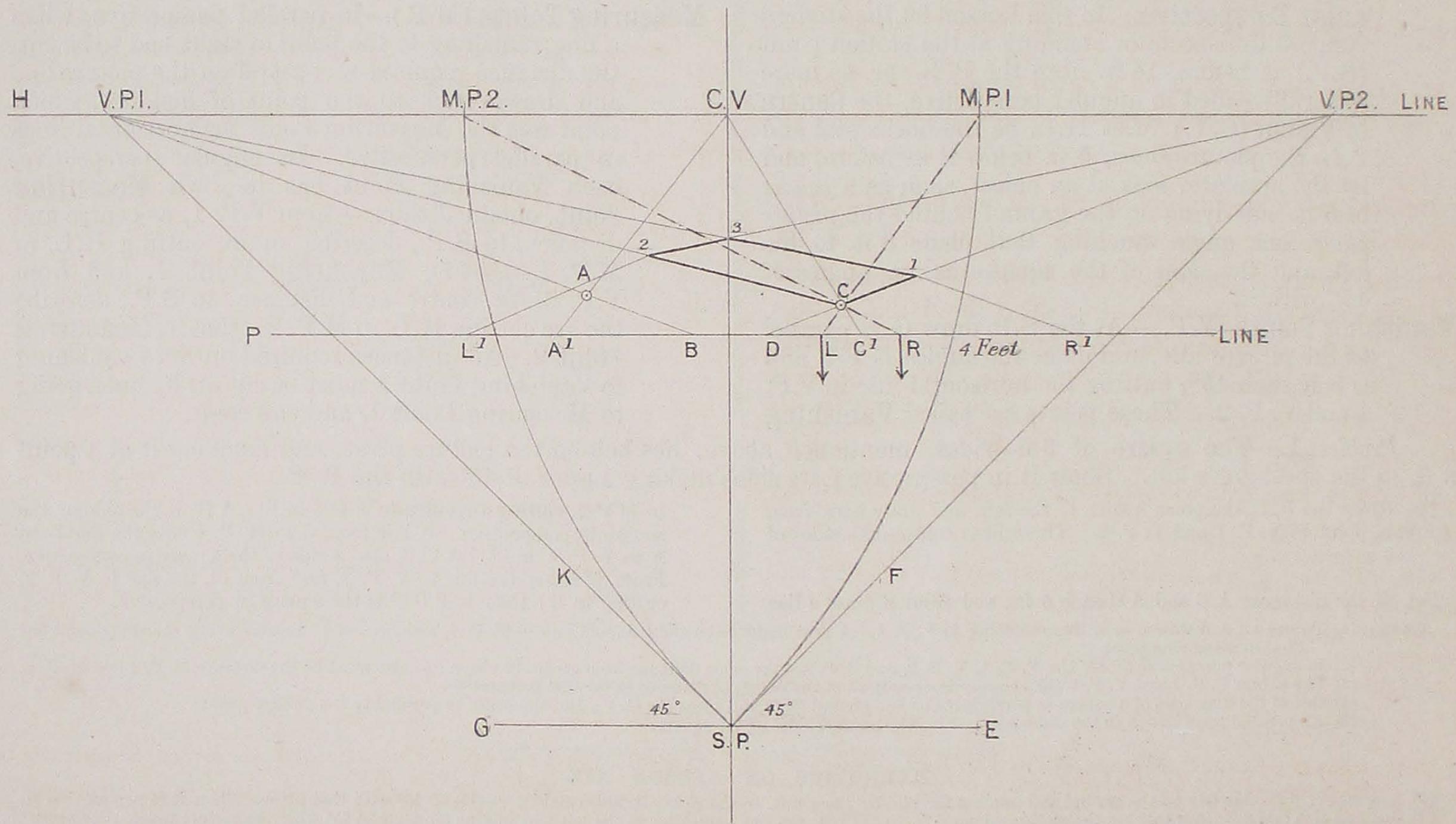
- (1) A square of 6 ft. side lies on the ground and touches the picture plane 5 ft. to the right, its sides making angles of 45° with that plane. Show it in perspective.
- (2) One end of a line 8 ft. long touches the picture plane 4 ft. to the left, the line vanishing to the left and making an angle of 45° with the picture plane. Represent it in perspective.

LESSON XX.

PLATE XX.

SCALE 48

45°



LESSON XX.

ANGULAR PERSPECTIVE, 45° .—(*The object not touching the picture plane.*)

Problem 1.—A point A is on the ground 4 ft. to the left of the spectator and 3 ft. within the picture—that is, behind the picture plane. Show that point in perspective.

The distances, height of eye, &c., as in the last Lesson.—Obtain the points V. P. 1, M. P. 1, &c., as in Lesson I.

NOTE.—To find a point within the picture in angular perspective proceed as in parallel perspective (always).

Problem 2.—A rectangle 8 ft. by 4 ft. lies on the ground behind the picture plane, its sides making angles of 45° with that plane, its nearest angle being 3 ft. to the right and 2 ft. within the picture—that is, behind the picture plane—the longer side vanishing to the left. Obtain the point C by setting off C¹, 3 ft. to the right and setting back on P. L. the distance C¹ D=2 ft. Join from C¹ to C. V. and from D to V. P. 2, cutting in C. This point is 3 ft. to the right and 2 ft. within the picture, as explained in Problem 1 above.

NOTE.—The student will notice that from points R and L the author has made this mark \downarrow . He has found that pupils frequently fail by taking their measurements from the wrong point, and hence advises them to use similar distinguishing marks, and remember, that all measurements required to the right must be set off from R towards the right and joined to M. P. 2, and all measurements on the left, from L and joined to M. P. 1, because it is required to cut lines vanishing to V. P. 1 and V. P. 2 respectively.

Set off on the P. L. the point A¹ 4 ft. to the left of the line of direction, and join A¹ to C. V. Set back the distance A¹ B, 3 ft., the distance the point A¹ is within the picture. Join B to V. P. 1, cutting the line from A¹ in A. This is the required point in perspective. This is the same as was done in finding a point within the picture in parallel perspective (Lesson III.), as the V. P. in this Lesson (for 45°) occupies the same position as the P. D. in that Lesson.

From M. P. 1 and M. P. 2 draw lines through C, cutting the Picture Line in L and R, to which a mark thus \downarrow is placed in the plate. Draw lines from C to V. P. 1 and V. P. 2: these lines contain the sides of the rectangle. From R set off on the P. L. the distance R R¹ 4 ft., and join R¹ to M. P. 2, cutting the line from C to V. P. 2 in point 1. C 1 is the 4 ft. side in perspective. From L set off on the P. line the distance L L¹=8 ft., and join L¹ to M. P. 1, cutting the line from C to V. P. 1 in point 2. C 2 is the 8 ft. side of the rectangle, and vanishes to the left at 45° . From points 1 and 2 draw the lines to V. P. 1 and V. P. 2, cutting each other in point 3. The figure C 1, 3, 2 is the rectangle in perspective.

EXERCISES ON LESSON XX.

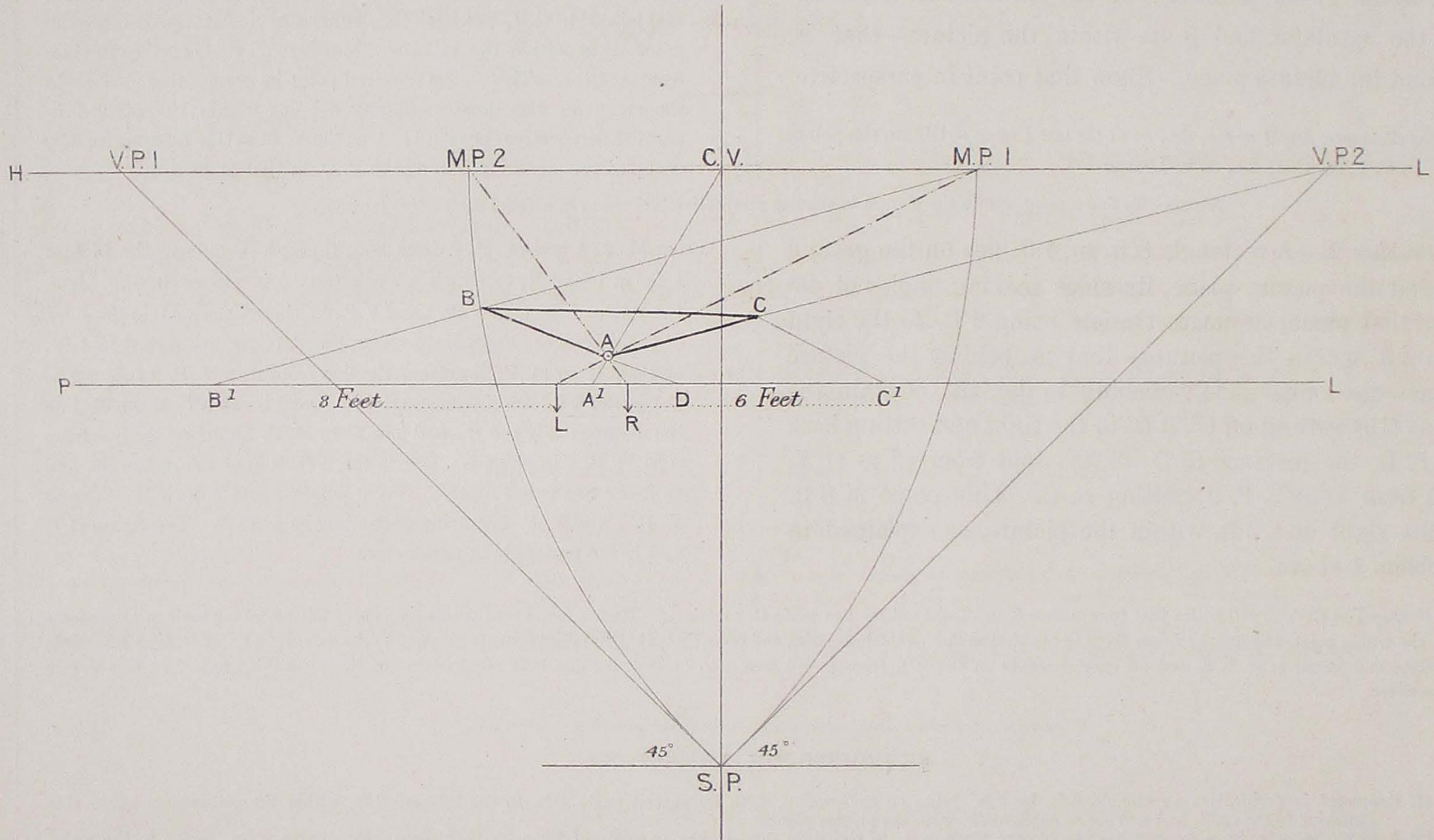
- (1) Represent in perspective a rectangle, 8 ft. by 6 ft., lying on the ground, with its nearest angle 2 ft. to the left and 3 ft. within the picture, its longer side vanishing to the right, and making an angle of 45° with the picture plane.
- (2) A rectangle, 6 ft. by 4 ft., touches the picture plane 4 ft. to the right, and makes an angle of 45° with that plane; its shorter sides vanish to the right. Represent it in perspective.

LESSON XXI

PLATE XXI.

SCALE $\frac{1}{48}$

45°



LESSON XXI.

ANGULAR PERSPECTIVE, 45° .—(*Height of eye, distance, etc., as before. Obtain V. P., M. P., etc., as in Lesson XIX.*)

Problem 1.—A right angled triangle, having its sides forming the right angle 6 ft. and 8 ft. long respectively, lies on the ground. Its nearest angle (the right angle) 3 ft. to the left and 2 ft. within the picture. Its sides containing the right angle make angles of 45° with the picture plane. Put it in perspective, when the longer arm of the right angle vanishes towards the left.

Find the point A, 3 ft. to the left and 2 ft. within the picture (Lessons III. and XX.) Draw lines to V. P. 1 and V. P. 2 from point A. Cut off L B¹, 8 ft., and R C¹, 6 ft., join to their respective measuring points, cutting the lines from A to V. P. in B and C. Join B C. A B C is the triangle required.

EXERCISES ON LESSON XXI.

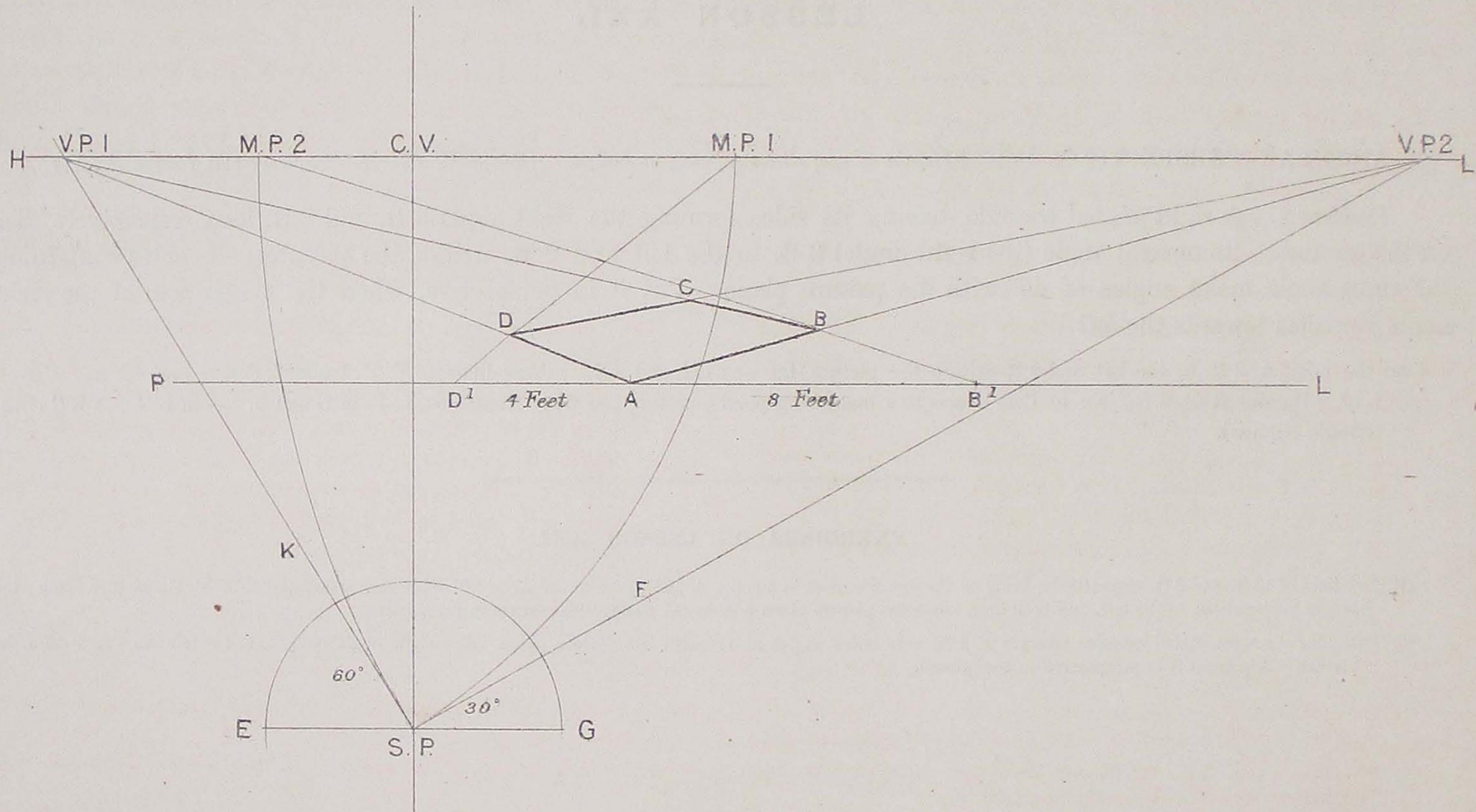
- (1) Two lines of 10 ft. and 8 ft. respectively, lying on the ground, meet in a point A and make an angle of 90° with each other and 45° with the picture plane ; the longer line vanishes to the left, and both lines touch the picture plane 6 ft. to the right. Place them in perspective.
- (2) Two sides of a right angled isosceles triangle of 6 ft. side make angles of 45° with the picture plane. The right angle is $2\frac{1}{2}$ ft. to the left and 1 ft. within the picture. Represent it in perspective on the ground.

LESSON XXII.

PLATE XXII.

SCALE $\frac{1}{48}$

30° & 60°



LESSON XXII.

ANGULAR PERSPECTIVE, 60° & 30° .—THE 30° BEING TO THE SPECTATOR'S RIGHT.—(*Distance, 13 ft.; height of eye, 5 ft.*)

In the line of direction from C. V., set off the station points S. P., 13 ft. Draw E G parallel to the horizontal line, and make the angle G P F, 30° , and E S K, 60° , producing the lines to cut the horizontal line in V. P. 2 and V. P. 1. From V. P. 1 as centre and distance to S. P., describe the arc cutting H. L. in M. P. 1, and from centre V. P. 2 and distance to S. P., the arc cutting H. L. in M. P. 2. These are the measuring points for their respective vanishing points. The Student will notice that V. P. 2 is farther from C. V. than in the previous Lessons (45°), and that V. P. 1 is nearer to C. V., because the angle that the object makes with the picture plane has altered from 45° , to 60° or 30° .

Problem 1.—A rectangle, 4 ft. by 8 ft., lies on the ground and touches the picture plane at a point 5 ft. to the right. Its longer side makes an angle of 30° with the picture plane, and vanishes towards the right. Represent it in perspective.

Set off on the P. L., the point A, 5 ft. to the right of the line of direction, and draw the lines to V. P. 1 and V. P. 2. Set off A B¹ 8 ft. to the right and A D¹ 4 ft. to the left, and join to M. P. 1 and M. P. 2 respectively, cutting the former lines in B and D. Draw

lines from B and D to the V. P. 1 and V. P. 2, cutting in C, then A B C D is the required rectangle in perspective.

Problem 2.—Given the perspective representation of the rectangle A B C D. Find the length of A B and A D. A line drawn from M. P. 2, through B to cut the picture line in B¹, gives A B¹ the length of the side A B. Also a line from M. P. 1, through D to cut the picture line in D¹, gives the length of the side A D. Measure the distances to the scale $\frac{1}{8}$, and it is found to be 8 ft. and 4 ft. long respectively. (This is the reverse of Problem 1).

EXERCISES ON LESSON XXII.

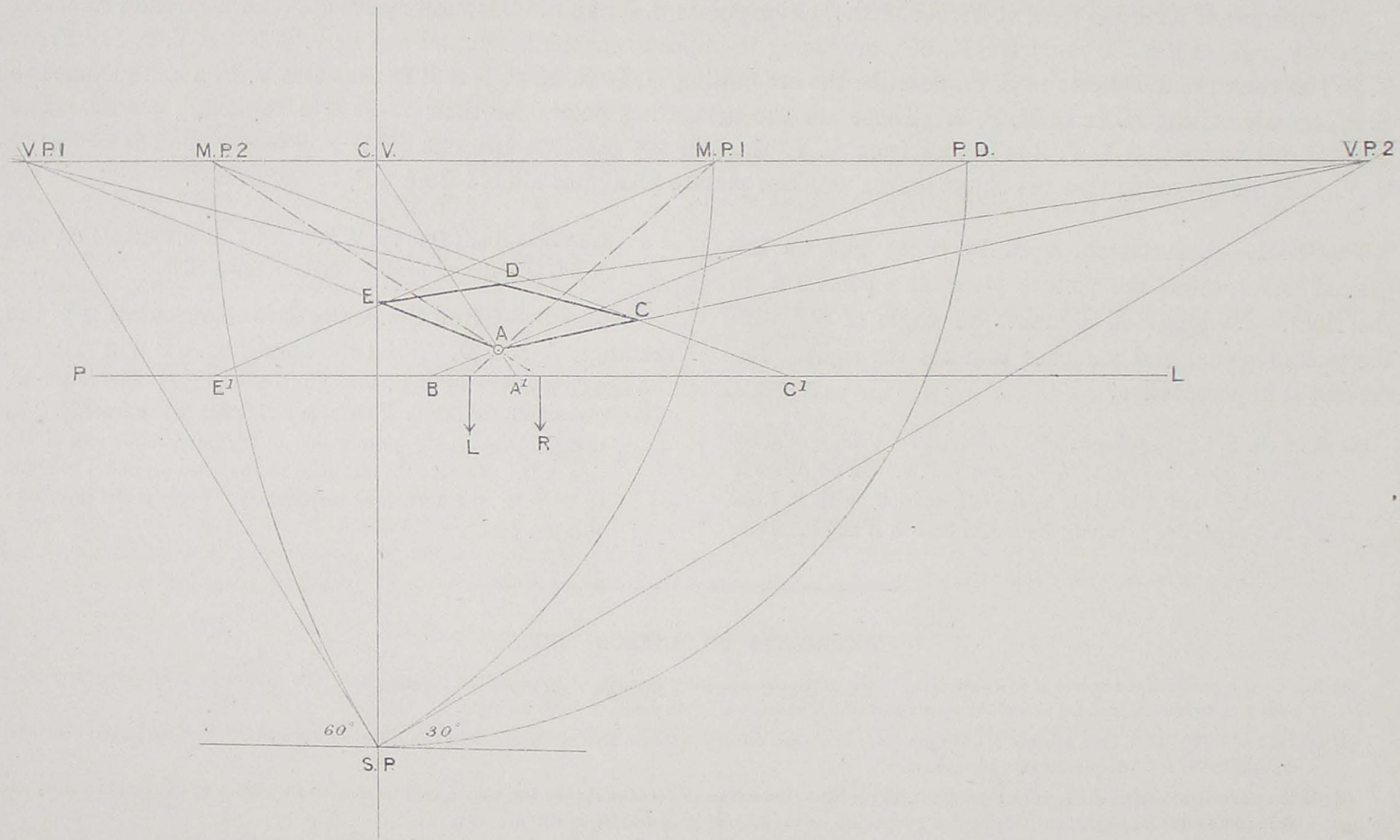
- (1) Represent in perspective a square of 4 ft. side, lying on the ground and making an angle of 60° with the picture plane. Its nearest angle touching the picture plane 3 ft. to the left, and the side of the square that makes an angle of 60° to vanish towards the right.
- (2) One end of a line, 10 ft. long, touches the picture plane at a point directly opposite the Spectator. The line makes an angle of 30° with the picture plane and vanishes to the left. Represent it in perspective.
- (3) On the picture line in Exercise I. mark a point 6 ft. to the left. Draw from this point a line to the vanishing point on the left. Mark off two real inches on this line and find its true length, *i.e.*, when out of perspective, to the scale of your drawing in that Exercise.

LESSON XXIII.

PLATE XXIII

SCALE 48

30° & 60°



LESSON XXIII.

ANGULAR PERSPECTIVE; 30° & 60° WITHIN THE PICTURE.—(*Distance, 14 ft.; height of eye, 5 ft.*)

As in the preceding Lesson, obtain Points V. P. 1 and V. P. 2; also M. P. 1 and M. P. 2.

Problem 1.—A square of 6 ft. side lies on the ground, and one side, on the right, makes an angle of 30° with the picture plane. Represent it in perspective when its nearest angle is $3\frac{1}{2}$ ft. to the spectator's right and 2 ft. within the picture.

First it is necessary to find the point $3\frac{1}{2}$ ft. to the right, and 2 ft. within the picture. This must always be found as explained in Lesson XX. Proceed thus:—set off on the picture line the point A^1 , $3\frac{1}{2}$ ft. to the right of the line of direction, and join A^1 to the centre of vision. From C. V. as centre and distance to S. P., describe the arc cutting the horizontal line in P. D., the point of DISTANCE (as in parallel perspective). From A^1 set back $A^1 B$ on the picture line, 2 ft., and join from B to the point P. D., cutting the line from A^1 in A. This point is where the angle of the square will fall, and is $3\frac{1}{2}$ ft. to the right and 2 ft. within the picture.

Draw lines from A to V. P. 1 and V. P. 2. From M. P. 1 draw a line through A to cut the P. L. in L, and a line from M. P. 2 through A to cut the P. L. in R. Set off on the P. L. the distance $R C^1$, 6 ft., and join from C^1 to M. P. 2, cutting the line from A to V. P. 2 in point C. A C is one side of the square. From L set off the distance $L E^1$, 6 ft., and join to M. P. 1, cutting the line from A to V. P. 1 in the point E. A E is another side of the square. From C and E draw lines to the respective vanishing points, cutting each other in D. Then A C D E is the required square in perspective.

Problem 2.—Suppose the point A, in Problem 1, to be given in perspective. Find its distance to the right, and within the picture.

Find the point of distance, as explained above, and draw lines from the C. V. and P. D. through A, to cut the picture line in A^1 and B. The distance from the line of direction to A^1 is its distance to the right, and the distance $A^1 B$ is its distance within the picture.

EXERCISES ON LESSON XXIII.

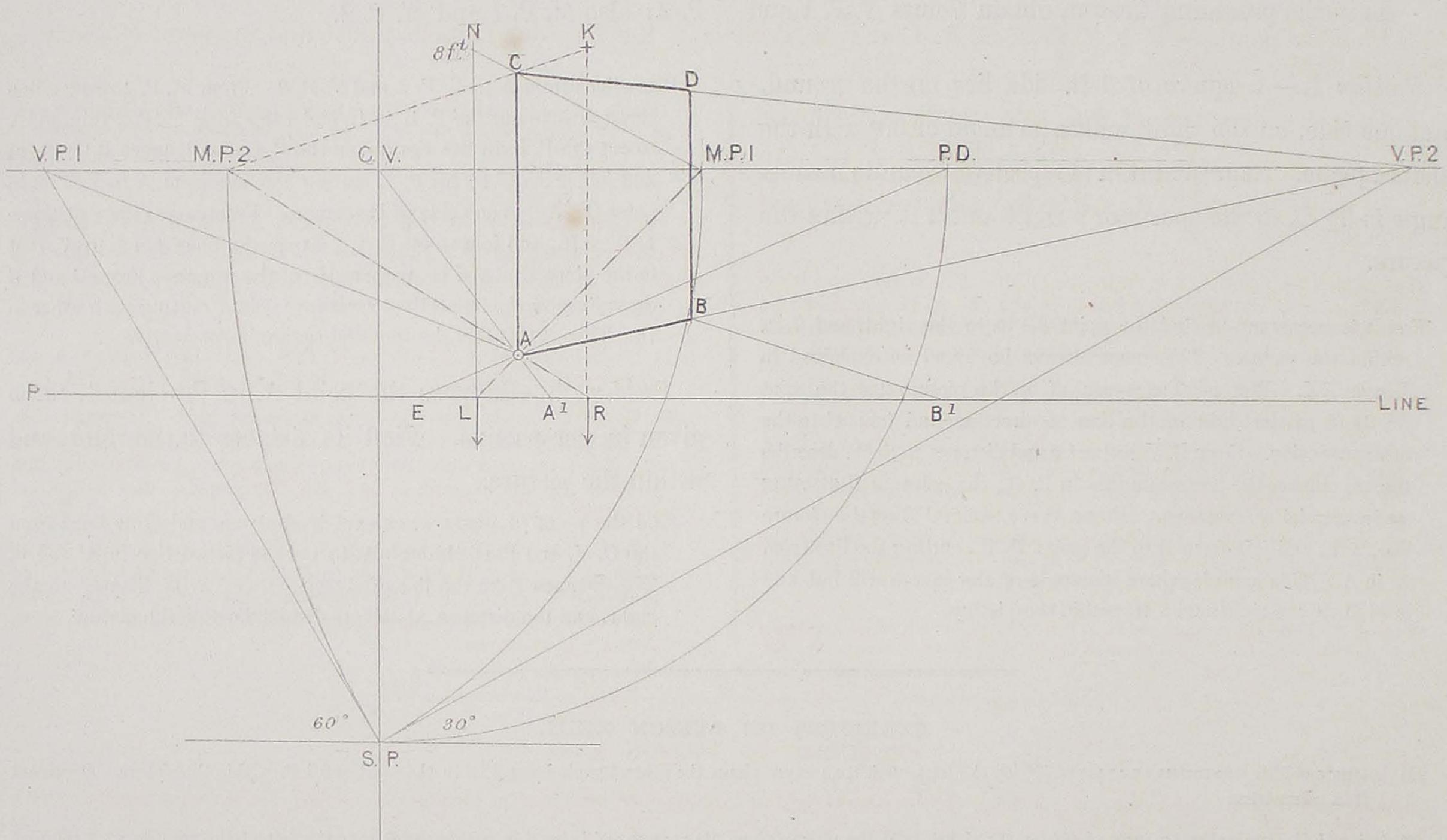
- (1) A square of 5 ft. side makes an angle of 30° (on the left), with the picture plane, the nearest angle being 2 ft. to the right and 1 ft. within the picture. Represent it in perspective.
- (2) A line 15 ft. long makes an angle of 60° (on the right), with the picture plane, its nearest end being 2 ft. within and 6 ft. to the left of the spectator. Represent it in perspective.

LESSON XXIV.

PLATE XXIV.

SCALE 48

30° & 60°



LESSON XXIV.

ANGULAR PERSPECTIVE, 30° & 60° —VERTICAL LINES & PLANES—(*Distance 13 ft., height of eye 5 ft., angles 30° & 60° .*).

Find the points V. P. 1, V. P. 2, M. P. 1, and M. P. 2, as in the previous Lessons; also find the point of distance, P. D., as before explained.

Problem 1.—A square of 8 ft. side stands vertically on the ground on one side, and makes an angle of 30° (on the right) with the picture plane, its nearest angle being 4 ft. to the right, and 3 ft. within the picture; represent it in perspective.

First obtain the point A, 4 ft. to the right, and 3 ft. within, and bring the lines from the measuring points cutting the P. Line in R and L; set off R B¹ 8 ft.; draw from B¹ to M. P. 2, cutting the line from A to V. P. 2 in B. A B is the side of the square on the ground.

To GET THE HEIGHT OF THE OTHER SIDE, A C. From any point in the *horizontal line* draw a line through A to the picture plane (*e.g.* L, R, A¹, or E); set up a perpendicular line from one of these points, as L N, 8 ft. high, (the length of the side of the square), and draw a line from N to the same point from which the foot of the perpendicular (L N) comes, (in this case from M. P. 1); draw A C, a vertical line, then A C is 8 ft. high in perspective; complete the square by drawing from C to V. P. 2, and B D perpendicular to cut it in D, then A C B D is the square in perspective.

NOTE—R K is set up 8 ft. high, and from both points (R and K) lines are drawn to M. P. 2, because the line from R through A cuts the horizon in that point; the

line from K will be found to pass through point C, proving that the height worked before is correct; so also a line 8 ft. high may have been set up perpendicular from point E or A¹, or any other point coming through A, the height, A C, always remaining the same.

Problem 2.—Given the two lines, A B and A C, in perspective; find the length of A B, height of A C, and the distance of the angle C A B within the picture, and to the spectator's right.

- (1) *To find length of A B.*—From M. P. 2, draw lines through A and B to cut the picture line in R and B¹; R B¹ is the length of A B.
- (2) *To find the height of A C.*—From any point in the horizontal line, as M. P. 2, draw lines through A and C, and from the point where the line through A cuts the picture line erect a perpendicular line to meet the line through C, as R K; this perpendicular is the real height of A C.
- (3) *To find the distance of the angle in the picture, and to the right.*—From C. V. and P. D. draw lines through A, cutting the picture line in A¹ and E, then A¹ E is the distance of the angle, C A B, within the picture, and the distance from the line of direction to A¹ is its distance to the right.

EXERCISES ON LESSON XXIV.

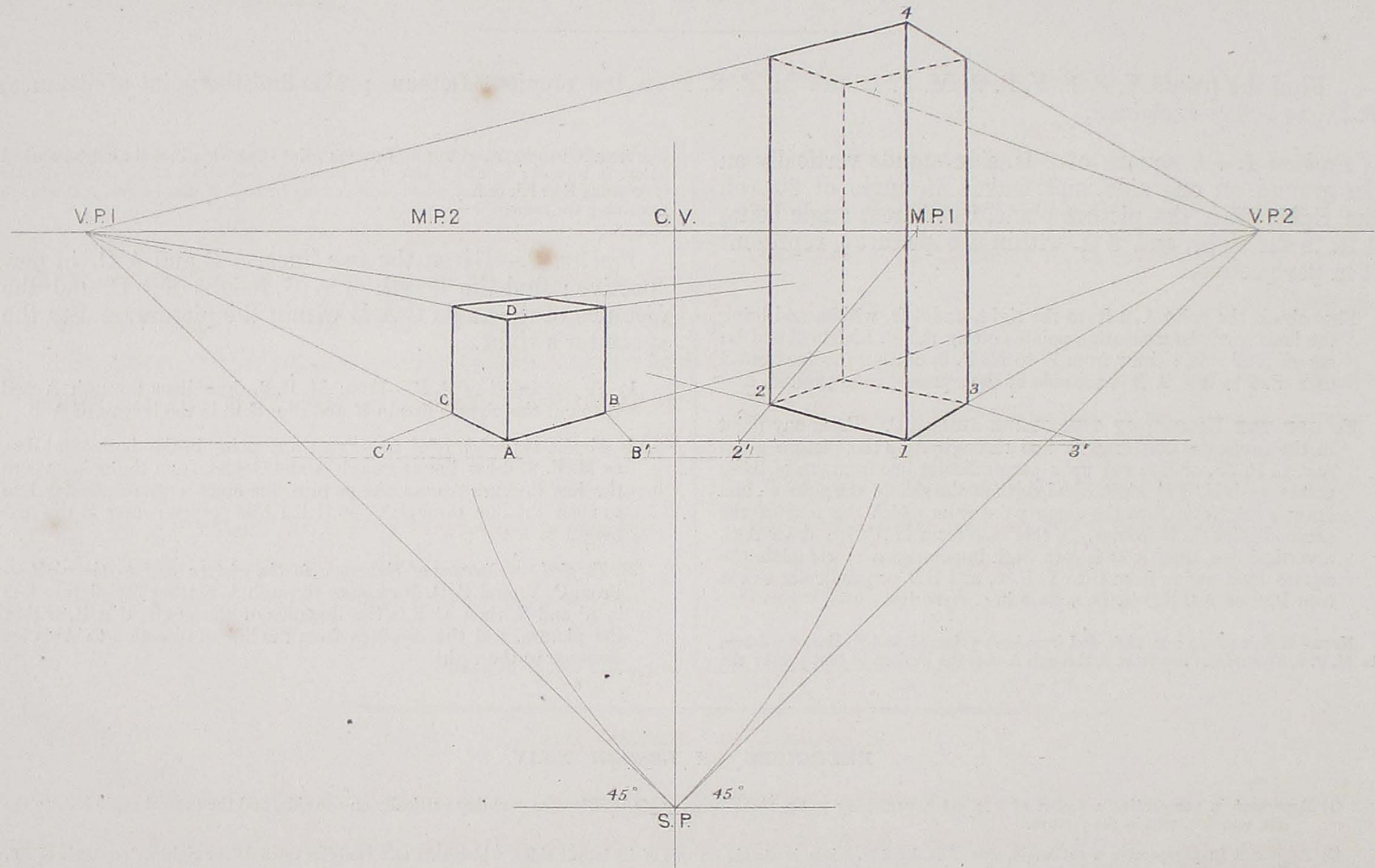
- (1) Represent in perspective a square of 8 ft. sides, vanishing to the right, at an angle of 60° when standing vertically on one side, and the nearest angle 5 ft. to the left, and 2 ft. within the picture.
- (2) Represent in perspective a rectangle, sides 8 ft. by 6 ft. when it stands on one of its longer sides, which side also vanishes towards the right at an angle of 30° , the nearest angle to be 2 ft. to the left, and 1 ft. within the picture.

LESSON XXV

PLATE XXV

SCALE $\frac{1}{48}$

45°



LESSON XXV.

ANGULAR PERSPECTIVE. 45°.—SOLIDS.—*Distance, 14 feet; height of eye, 5 feet; angles 45°.*

Problem 1.—A cube of 3 ft. edge touches the picture plane 4 ft. to the left. It rests on one face, and another face vanishes to the right at an angle of 45°. Represent it in perspective.

First find the V. P. 1, V. P. 2, M. P. 1, and M. P. 2 as explained in previous Lessons. Set off point A=4 ft to the left and draw lines to the vanishing points from it. Set off A B¹ and A C¹ each 3 ft., and obtain the points B and C by drawing to M. P. 1 and M. P. 2. Since the cube touches the picture plane and vanishes at an angle, it is evident that one edge touches the picture plane. Set up A D on the picture plane, from A, 3 ft. high. This is the edge touching

the picture plane, draw lines from D to the vanishing points, and complete as shown.

Problem 2.—A rectangular block with a square end rests on its end, on the ground. Each oblong face of the block is 10 ft. by 4 ft. Represent it in perspective, when one edge touches the picture plane 5 ft. 6 in. to the right; the shorter edges making angles of 45° with the picture plane.

Set off point 1, 5 ft. 6 in. to the right and obtain points 2 and 3 in perspective, as explained for B and C, Problem 1, above. Set up the height 1 to 4, 10 ft., this is the edge touching the P. P., and complete as shown in the figure.

EXERCISES ON LESSON XXV.

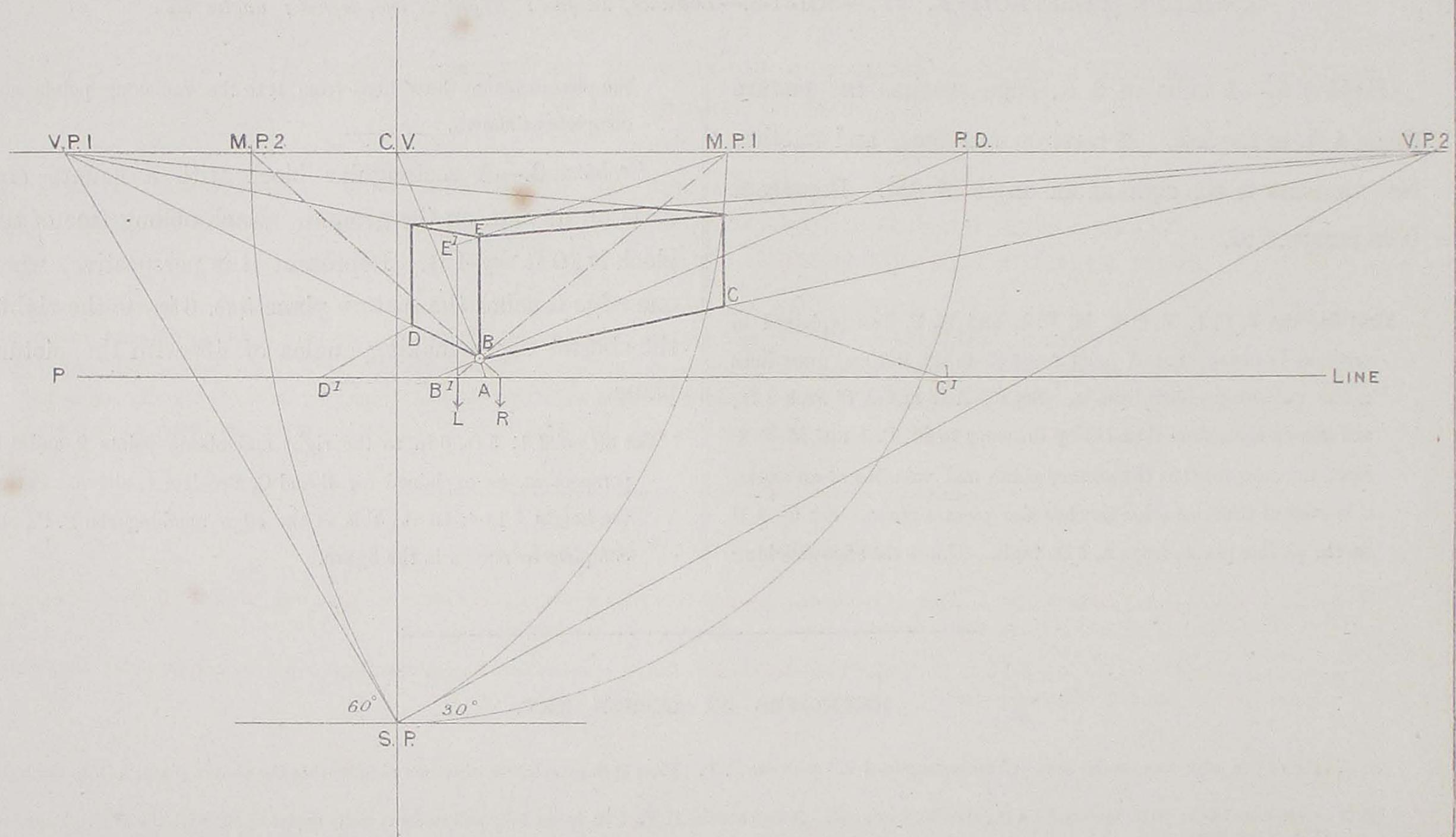
- (1) A cube of 6 ft. edge rests on one face, and makes angles of 45° with the P. P. Place it in perspective when one edge touches the picture plane, 3 ft. to the right.
- (2) A rectangular block, 10 ft. by 6 ft. by 4 ft., stands on one end. It touches the P. P., 5 ft. to the left, and its faces make angles of 45° with the P. P. Represent it in perspective.

LESSON XXVI.

PLATE XXVI.

SCALE $\frac{1}{48}$

60° & 30°



LESSON XXVI.

ANGULAR PERSPECTIVE.—SOLIDS.—RECTANGULAR BLOCKS WITHIN THE PICTURE.—(*Distance, 13 ft. ; height of eye, 5 ft.*
Scale $\frac{1}{8}$ s. Angles 60° and 30° .

Problem 1.—A rectangular block, 10 ft. long with a square end of 3 ft. edge, rests on the ground on one of its rectangular faces. Its longer edges vanish to the right at an angle of 30° . Represent it in perspective when the nearest corner is 2 ft. to the right and 1 ft. within the picture.

Having found the V. P.'s, P. D., and M. P.'s, picture line, &c., find the point B 2 ft. to the right and 1 ft. within the picture (as in Lesson III). Join to the V. P.'s and obtain points C and D, the one 10 ft. from B and the other 3 ft. from B, taking great care to measure them from points R and L (right and left) respectively, as explained in Lesson 20. From the end of any line passing through point B and meeting the picture line, such as the line from M. P. 1 to L, set up the height E¹, 3 ft. Join to the same point that the line through B comes from, in the horizontal line, (in this case M. P. 1) then B E is 3 ft. in perspective. From E draw lines right and left to the V. P.'s, and complete as shown in the figure.

EXERCISES ON LESSON XXVI.

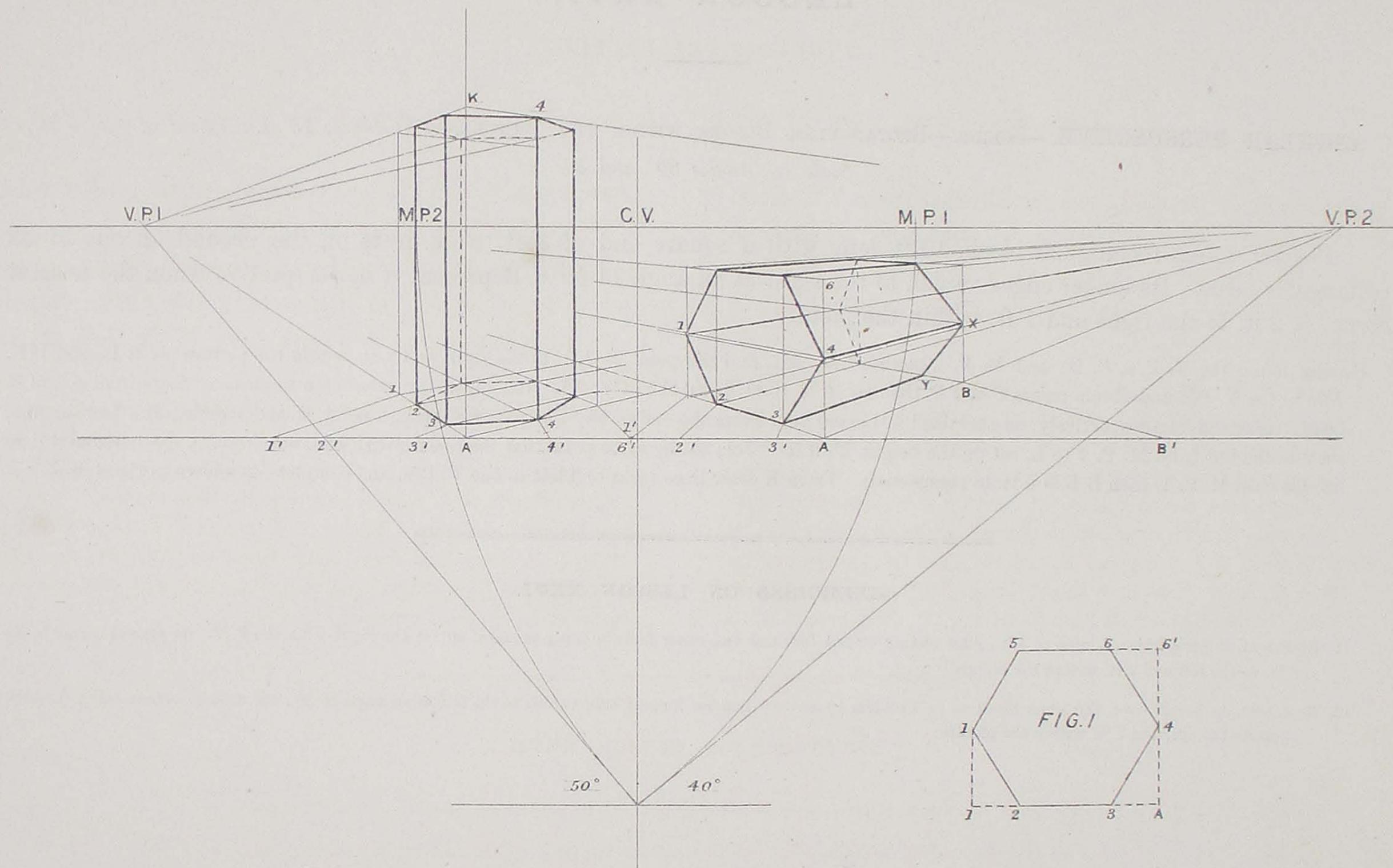
- (1) Represent, in perspective, a cube of 4 ft., edge resting on one face and two other faces making angles of 60° on the right with the P. P., its nearest corner being 5 ft. to the left and 2 ft. within the picture.
- (2) Represent, in perspective, the same block as in Problem 1 (above) when its longest side vanish to the left at an angle of 30° , its nearest corner being directly opposite the eye, but 2 ft. within the picture.

LESSON XXVII.

PLATE XXVII.

SCALE $\frac{1}{48}$

40° & 50°



LESSON XXVII.

ANGULAR PERSPECTIVE.

HEXAGONAL PRISM, Angles 50° and 40° . *Distance, 14 ft.; height of eye, 5 ft.*

Problem 1 (on the right).—A hexagonal prism 8 ft. long. Its end, shown in Fig. 1, is to be put in perspective so that point 4 shall be 4 ft. 6 in. to the right, and touch the picture plane. The axis of the prism to vanish to the right at an angle of 40° .

Find the V. P.'s, M. P.'s, etc., for the angles of 50° and 40° , so that 40° shall be on the right. Set off on P. Line the point A, 4 ft. 6 in. to the right, and draw lines to V. P. 1 and V. P. 2; set off from A the points A 3¹, A 2¹, A 1¹ to the left of it on the P. L. corresponding to the distance A 3, 2, 1 of Fig. 1; set off A B¹ to the right 8 ft., draw lines to the respective measuring points, to cut the lines to V. P. 1 and V. P. 2 in points B, 3, 2, etc.; set up A 4 equal to A 4 (Fig. 1), and 4, 6 the same height, join to V. P. 1, and complete the end by perpendicular lines from points 2 and 3, etc. From 3 and 4

draw lines to V. P. 2 to meet the line from B; the points X Y are obtained; so for the other points. Complete as shown.

Problem 2 (on the left).—The same prism, standing on one end, the point A, Fig. 1, touching the picture plane 4 ft. to the left, and one side vanishing to the left at an angle of 50° .

Set off A, 4 ft. to the left; draw lines to V. P.'s; set off points A 3¹, 1¹ on the left, and A 4¹ and A 6¹ on the right equal to the corresponding points in Fig. 1; draw to their respective, M. P.'s to obtain points 3, 2, 1, 4, etc.; complete the base as shown.

To get the height—Set up A K, 8 ft. high, join to V. P. 1 and V. P. 2; these lines correspond with similar lines from A on the ground; perpendicular lines drawn from points 4, 3, 2, etc., will give the upper end; complete as shown.

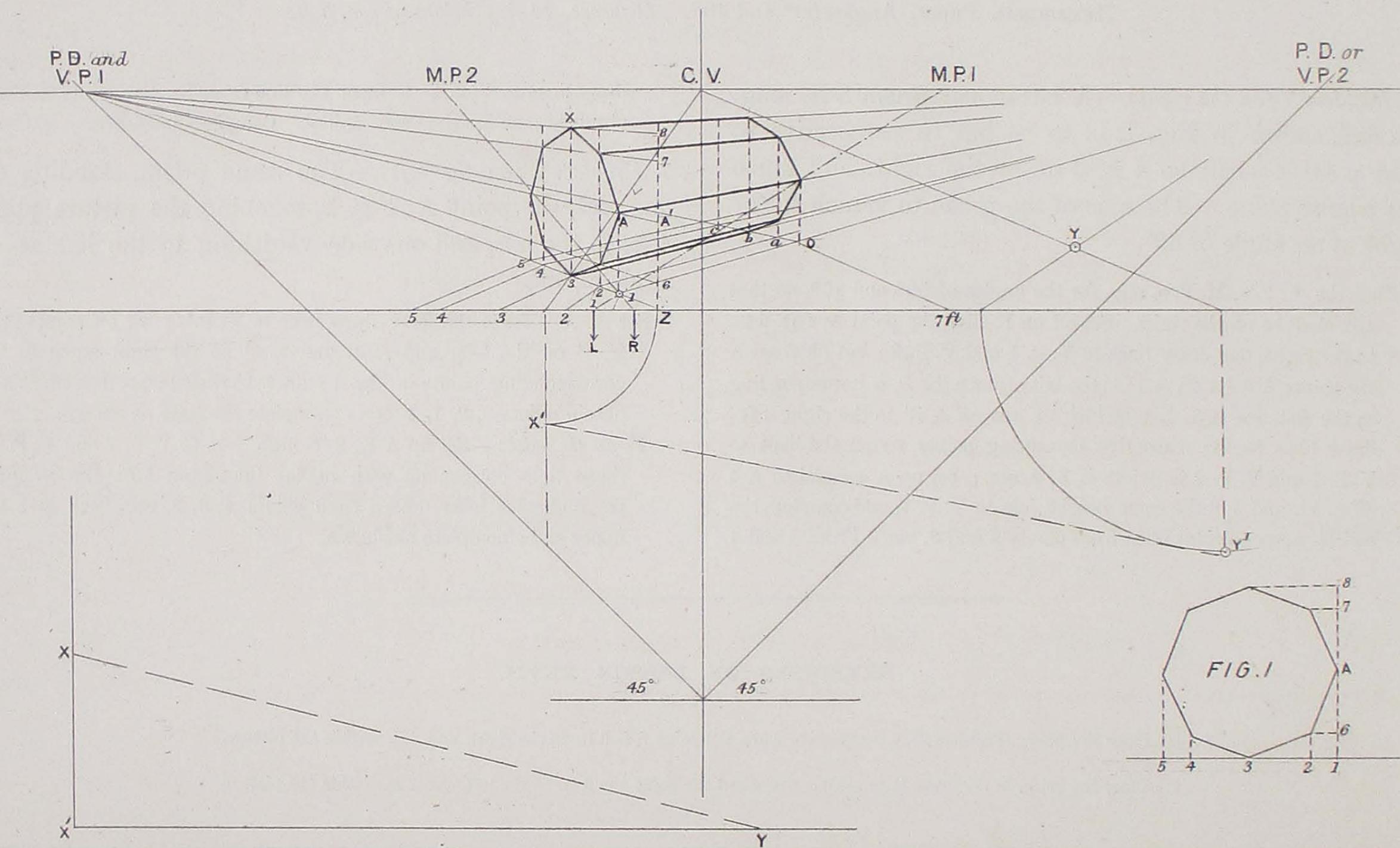
EXERCISES ON LESSON XXVII.

- (1) Place the prism (Problem 2) in perspective when the point A is 3 ft. to the right, and 2 ft. within the picture.
- (2) Place the prism in (Problem 1) in perspective when the point 4 is 6 ft. to the left, and 2 ft. within the picture.

LESSON XXVIII.
OCTAGONAL PRISM-ANGLE 45°

PLATE XXVIII.

SCALE 48



LESSON XXVIII.

OCTAGONAL PRISM.—ANGLES 45° .—ANGULAR PERSPECTIVE.—*Distance, 14 ft.; height of eye, 5 ft.; angles, 45° ; scale, $\frac{1}{8}$.*

Problem 1.—An octagonal prism 7 ft. long. Its end, shown in Fig. 1, is to be placed in perspective, its axis vanishing to the right at an angle of 45° , and the point A to be 2 ft. to the left and 1 ft. within the picture.

Obtain the V. P.'s, M. P.'s, C. V., etc., as explained before. Set off on the picture line, 2 ft. to the left, and obtain the point \odot on the ground, 2 ft. to the left, and 1 foot within as explained in Lesson III., parallel. Draw the lines L and R from the M. P.'s, and set off from L on the picture line the points 1, 2, 3, 4, 5, corresponding to those in Fig. 1; draw to M. P. I, cutting the line through \odot to V. P. I; set up $Z A^1 = 1 A$, Fig. 1; $Z 6 = 1, 6$, Fig. 1, etc., and join each point to the same point as the line from Z is drawn (V. P. I), cutting the perpendicular lines raised from points 1, 2, 3, 4, 5, in the line Z, V. P. I; complete the end as shown.

Set off from R, 7 ft., and join to M. P. 2, cutting the line from point

\odot in O. Join O to V. P. 1, and draw lines from 1, 2, 3, 4, 5, on the line Z, V. P. 1, to V. P. 2, cutting the former line in a, b, c, etc. Erect perpendicular lines from these points to cut lines from the angles of the base to V. P. 2, and complete as shown on the plate.

Problem 2.—Find the length of a string to reach from point Y on the ground to point X on the prism.

Take the point Y out of perspective by drawing lines from P. D. and C. V. through Y as explained before. Do the same for point X, first taking it to the ground (3). X^1 and Y^1 are the two points on the ground. The point X is perpendicularly above the point X^1 , because X is perpendicularly above point 3 in perspective. Hence the right-angled triangle $X Y X^1$, where the height $X^1 X$ is the height $Z 8$, or 1 8 Fig. 1, gives the length of the string required. It is $X Y$, the hypotenuse of the right-angled triangle, which measured to the scale $\frac{1}{8}$ is just 16 ft. 6 ins.

EXERCISES ON LESSON XXVIII.

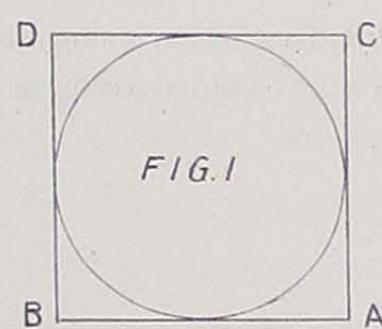
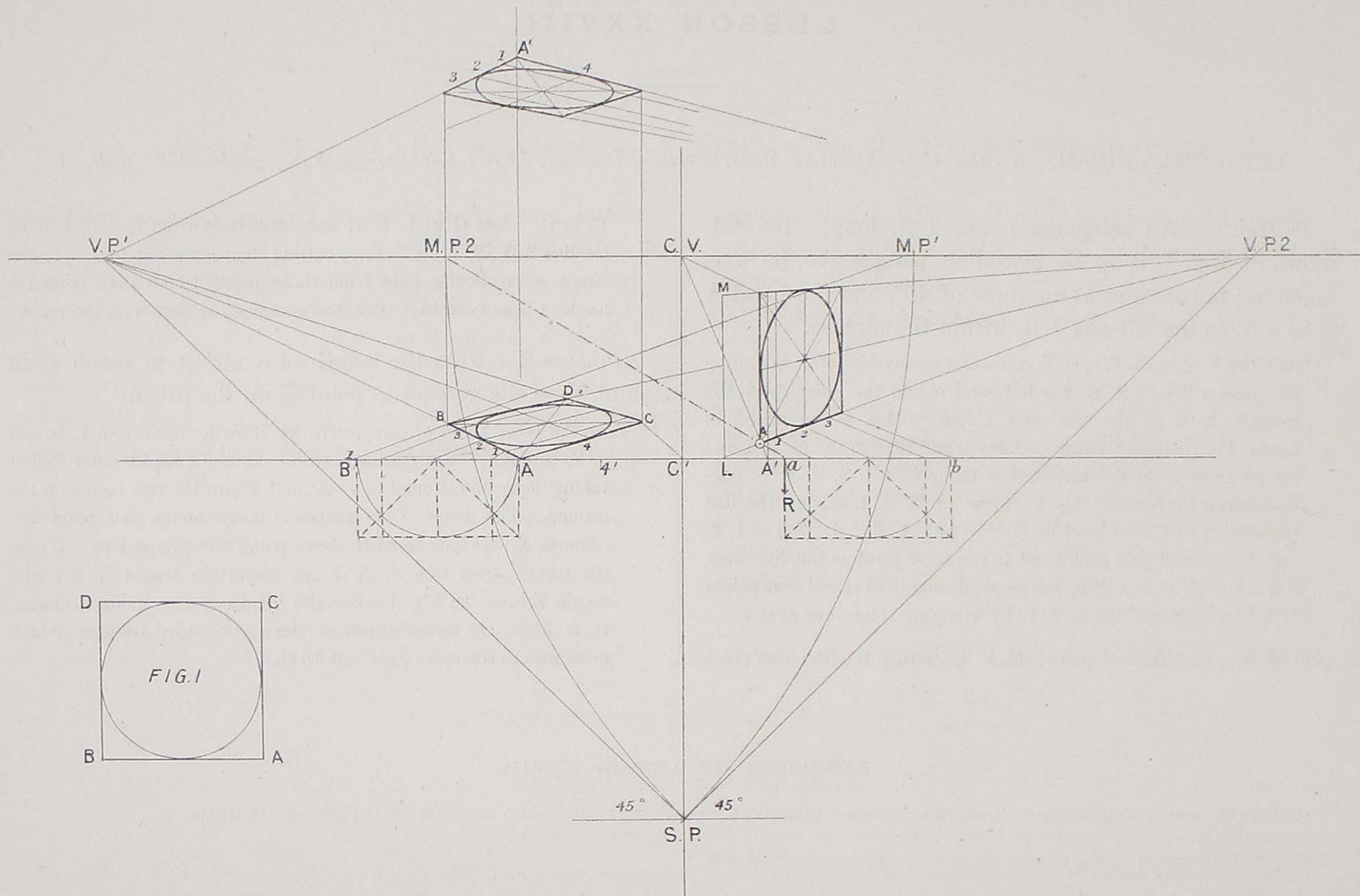
- (1) Place the same prism in perspective when its axis vanishes to the left at an angle of 45° , the point A being 3 ft. to the right and 2 ft. within.
- (2) Find the length of a string to reach to two similar points, as in Problem 2.

LESSON XXIX

PLATE XXIX

SCALE $\frac{1}{48}$

45°



LESSON XXIX.

ANGULAR PERSPECTIVE.—CIRCLES.—(*Distance, 14 ft. ; height of eye, 5 ft. ; angles, 45° ; scale, $\frac{1}{8}$.*)

Problem 1 (on the left).—Fig. 1. A B C D is a square with a circle inscribed in it. Place both in perspective, on the ground, when the point A is touching the picture plane 4 ft. to the left, and A C vanishes to the right at an angle of 45°.

Obtain the V. P.'s, &c., as before explained. Set off A on the picture line, 4 ft. to the left, and $A B^1 = A B$ Fig. 1. Join A to V. P. 1 and V. P. 2, and set off $A C^1 = A C$ (Fig. 1) on the picture line. Describe the semicircle on $A B^1$ and surround it as shown. Draw lines from each point in $A B^1$ to M. P. 1, cutting the line from A to V. P. 1 in 1 2 3 B. Draw from C^1 to M. P. 2, cutting lines from A to V. P. 2 in C. Complete the square A B C D. Join the diagonals and draw the lines from 1 2 3 4 to V. P. 2, cutting the diagonals. Draw the ellipse through those points.

Problem 2.—Place the same figure, in perspective,

parallel to the ground, but 10 ft. high, its sides vanishing as in Problem 1.

Set up $A A^1$, 10 ft. high, and join to V. P. 1 and V. P. 2. The points 1 2 3 4 are vertically over corresponding points in *Problem 1*. Complete as in the figure.

Problem 3.—Place the same figure, in perspective, when perpendicular to the ground, the point A being 2 ft. to the right and 1 ft. within the picture; it vanishes to the right at an angle of 45°.

Find the point A, 2 ft. to the right and 1 ft. within the picture. Draw a line from M. P. 2 through A to the picture line in point A, and join A to V. P. 2. Draw the semicircle, and obtain the points 1 2 3, etc., in this line A, V. P. 2. Set up the height from any line through A, as L M equal the side of the square, and join back to the same point in the horizon (V. P. 2 in this case). Complete as shown.

EXERCISES ON LESSON XXIX.

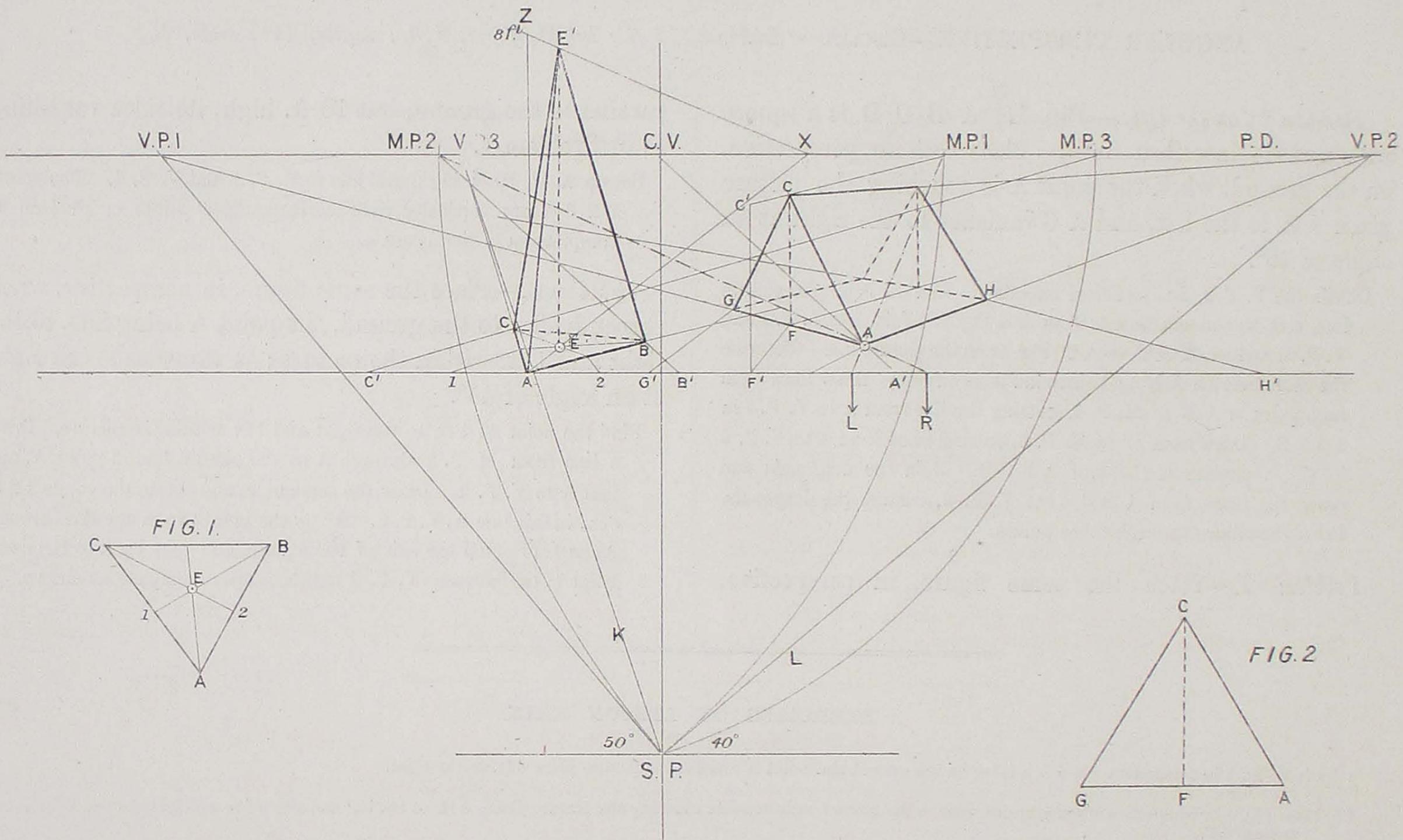
- (1) Place Fig. 1 in perspective when it is lying on the ground and point A touches the picture plane 3 ft. to the right.
- (2) Place Fig. 1 in perspective when perpendicular to the ground with one side touching the picture plane, 5 ft. to the left and two sides vanishing to the right at an angle of 45°.

LESSON XXX.

PLATE XXX.

SCALE $\frac{1}{48}$

40° & 50°



LESSON XXX.

ANGULAR PERSPECTIVE.

TRIANGULAR, PYRAMID, AND PRISM.—(*Distance, 14 ft. ; height of eye, 5 ft. ; scale, $\frac{1}{8}$; angles, 50° & 40° .*)

Problem 1.—Fig. 1 is the plan of a triangular pyramid 8 ft. high. Place it in perspective when standing on its base on the ground and point A touches the picture plane 3 ft. to the left, and A B vanishes to the right at an angle of 40° .

Obtain points V. P. 1 and 2, M. P.'s, etc., as before :

Set off A, 3 ft. to the left, and draw A to V. P. 2 ; bisect each angle of the triangle in Fig. 1, to get the centre E ; set off A B¹ on the picture line=A B, Fig. 1, and draw to M. P. 2, cutting the line from A in point B ; A B is the side of the triangle (Fig 1 A B) in perspective.

TO OBTAIN THE SIDE, A C.—A C is not at right angles to A B, so does not vanish to the V. P. 1 ; the vanishing point is V. P. 3, obtained thus :—make the angle to L, P, K, equal the angle at A, Fig. 1, then S. P. K cuts the horizontal line in V. P. 3 ; from centre V. P. 3 and radius to S. P., describe the arc cutting the horizon in M. P. 3, the measuring point for V. P. 3.

Draw from A to V. P. 3 ; set off on the picture line A C¹ = A C, Fig. 1 ; draw from C¹ to M. P. 3, cutting the line from A in C ; A C

is the other side of the triangle in perspective ; join B C by the dotted line. Find the centre of the triangle thus :—set off A 1 and A 2 on the picture line equal to A 1 A 2, Fig. 1 ; join to their respective M. P.'s to cut the sides of the triangle, and join to its angles, cutting in E ; E is the centre of the triangle. Set up from A, the perpendicular line A Z, 8 ft. high, the height of the pyramid ; draw a line from A through E to cut the horizon in X, and join Z X ; E E¹ is the 8 ft. in perspective, join to the angles, A, B, and C, as shown.

Problem 2.—Fig. 2 is the end of a prism 8 ft. long. Place it in perspective when it rests on one face and its edges vanish to the right at an angle of 40° , so that the point A shall be $5\frac{1}{2}$ ft. to the right, and 2 ft. within the picture.

Obtain the point A as required, and draw A H and A G to the V. P. 1 and V. P. 2 ; bring the lines from M. P.'s through A to cut the picture line in L and R ; set off L F¹ G¹ = F G (Fig. 2) ; join to M. P. 1, cutting A G in F and G ; set off R H¹ = 8 ft., and draw H¹ to M. P. 2, cutting A H in H ; set up F¹ C¹ = F C (Fig. 2) ; join to M. P. 1, because F¹ F goes to that point, F C is the height in perspective. Join C G and C A ; draw lines from H to V. P. 1, and from F and G to V. P. 2, and complete as shown in the figure.

EXERCISES ON LESSON XXX.

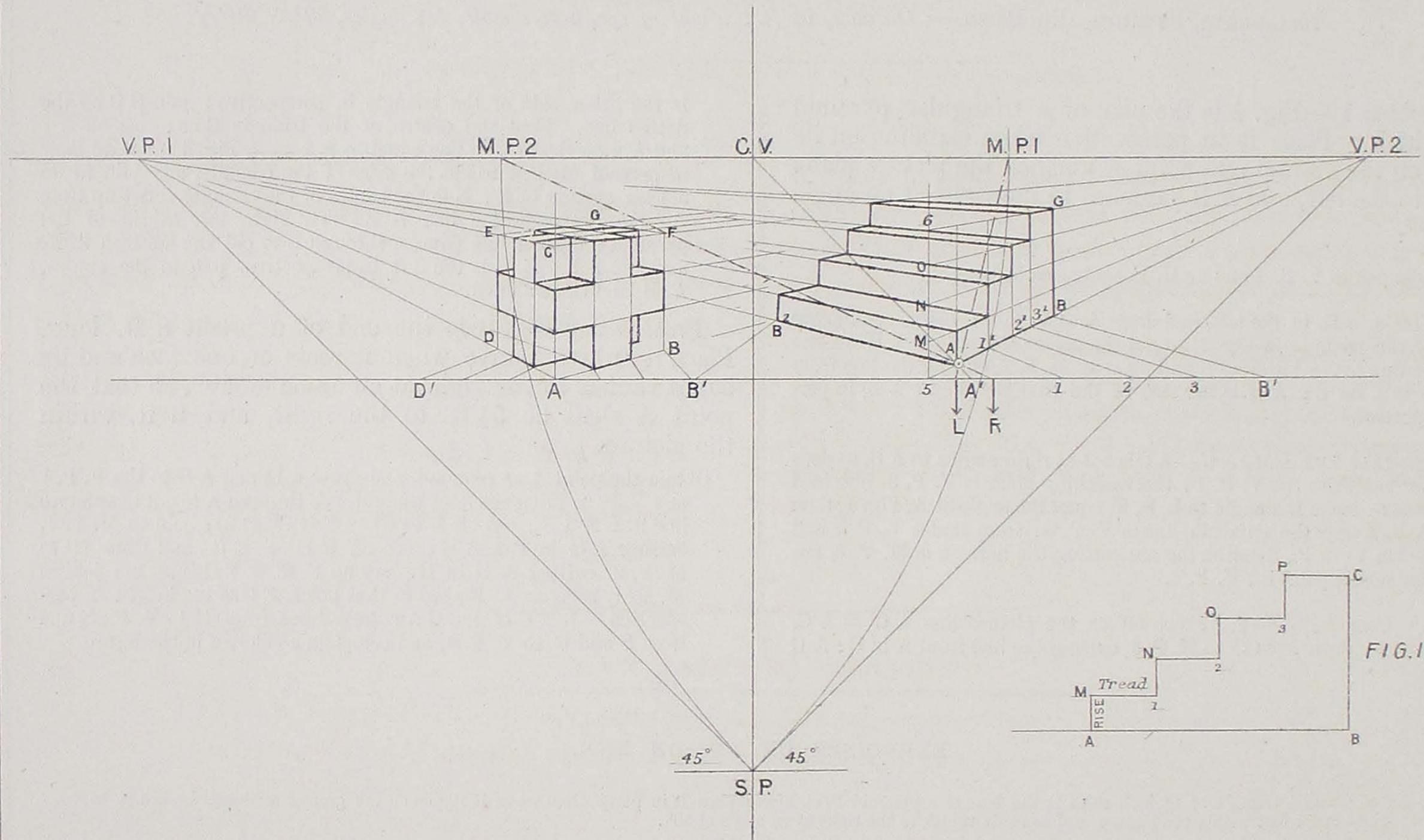
- (1) An equilateral triangle of $1\frac{1}{2}$ inch sides is the base of a pyramid 10 ft. high ; place it in perspective when it stands on the ground with one angle 4 ft. to the right and 1 foot within the picture, and one side vanish to the right at an angle of 40° .
- (2) Place the prism (in Problem 2 above) in perspective, when the point A touches the picture plane 5 ft. to the left, and A H vanishes to the right at an angle of 40° .

LESSON XXXI.

PLATE XXXI.

SCALE 48

45°



LESSON XXXI.

ANGULAR PERSPECTIVE.

(Distance, 14 ft.; height of eye, 5 ft.; scale, $\frac{1}{8}$; angle, 45° .)

Problem 1 (on the left).—Out of a solid block of stone of 3 ft. edge, a cross, one foot arm, is cut on each face. Place it in perspective with four faces vanishing at 45° , and one edge touching the picture plane $4\frac{1}{2}$ ft. to the left.

Obtain V. P.'s and M. P.'s, etc., as before.

Find point A, $4\frac{1}{2}$ ft. to the left, and put the cube, A B C D E F G in perspective (as in Lesson VII); divide A D¹, A B¹, and A C into three parts, each is 1 foot; draw lines to the measuring points to cut A D and A B; complete the solid cross as shown.

Problem 2 (on the right).—Fig. 1 is the end of a flight of steps 6 ft. wide. Place them in perspective when the edges of the steps vanish to the left at an angle of 45° , and the point A is 5 ft. to the right and 1 ft. within the picture.

Obtain the point A as required, and bring the line from M. P. 1 and M. P. 2 to L and R; set off the distance, R 1, 2, 3, B¹, in the picture plane, equal to the tread of the step, M 1, N 2, etc.; draw to M. P. 2 to cut the line from A in B 1¹, 2¹, etc.; set off L B¹ = 6 ft., and draw to M. P. 1, cutting A B in B²; set up the perpendicular line 5 6 = to B C (Fig 1), and mark off the rise, M, N, O, on it; draw lines to V. P. 2, meeting perpendicular lines from 1¹ 2¹ 3¹ B, and complete as shown.

EXERCISES ON LESSON XXXI.

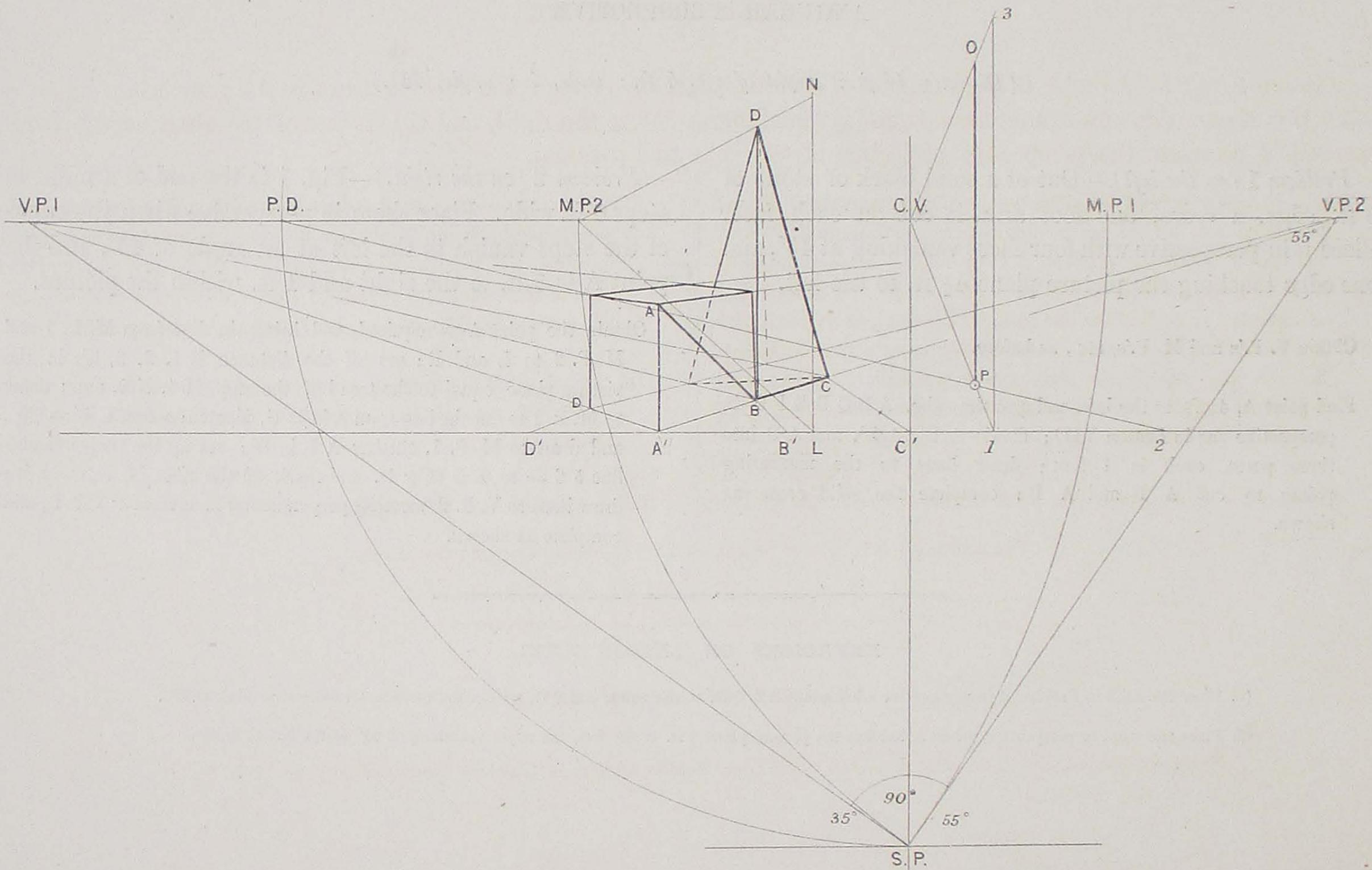
- (1) Place the solid in Problem 1 in perspective when point A is 5 ft. to the right, and 2 ft. within the picture, its sides vanishing at 45° .
- (2) Place the steps in perspective, when A touches the picture plane 2 ft. to the left, the edges vanishing at 45° to the left as before.

LESSON XXXII.

PLATE XXXII.

SCALE $\frac{1}{48}$

35° & 55°



LESSON XXXII.

ANGULAR PERSPECTIVE.

Problem 1 (on the left.)—Given A B, the diagonal of a vertical face of a cube ; the face vanishes to the right at an angle of 55° , B C is one edge of a square base pyramid vanishing at 55° to the right, and C D is one of the slant edges of that pyramid ; complete the perspective projection of the cube and pyramid.

The only data that need be given besides the above is the C. V. and horizontal line, the rest can be obtained thus :—through C. V. draw the line of direction ; produce B C to cut the horizontal line in V. P. 2 ; make the angle at V. P. 2 = 55° , and the line cuts the line of direction in S. P. ; the line to V. P. 1 is at right angles to this line from V. P. 2 ; obtain necessary points as explained before. Since A is on the picture plane, the perpendicular, A A¹, meeting B C, produced to A, gives a point in the picture line, which draw parallel to the horizontal line.

Complete thus :—from M. P. 2 draw lines through B and C, cutting the picture line in B¹ and C¹, then A¹ B¹ is the true length of the edge of the cube, and is equal to A¹ A, and B¹ C¹ is the true length of the edge of the base of the pyramid ; complete the cube as shown, and join from point D to each angle of the base of the pyramid.

If the height of the pyramid is required, draw lines from any point, as M. P. 2, through the centre of the base and point D, then L N is the true height.

Problem 2.—Required, the true height of the line O P given in perspective on the right.

The distance 1 3 is its true height, and 1 2 its distance within the picture.

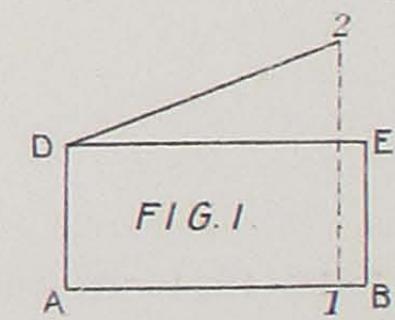
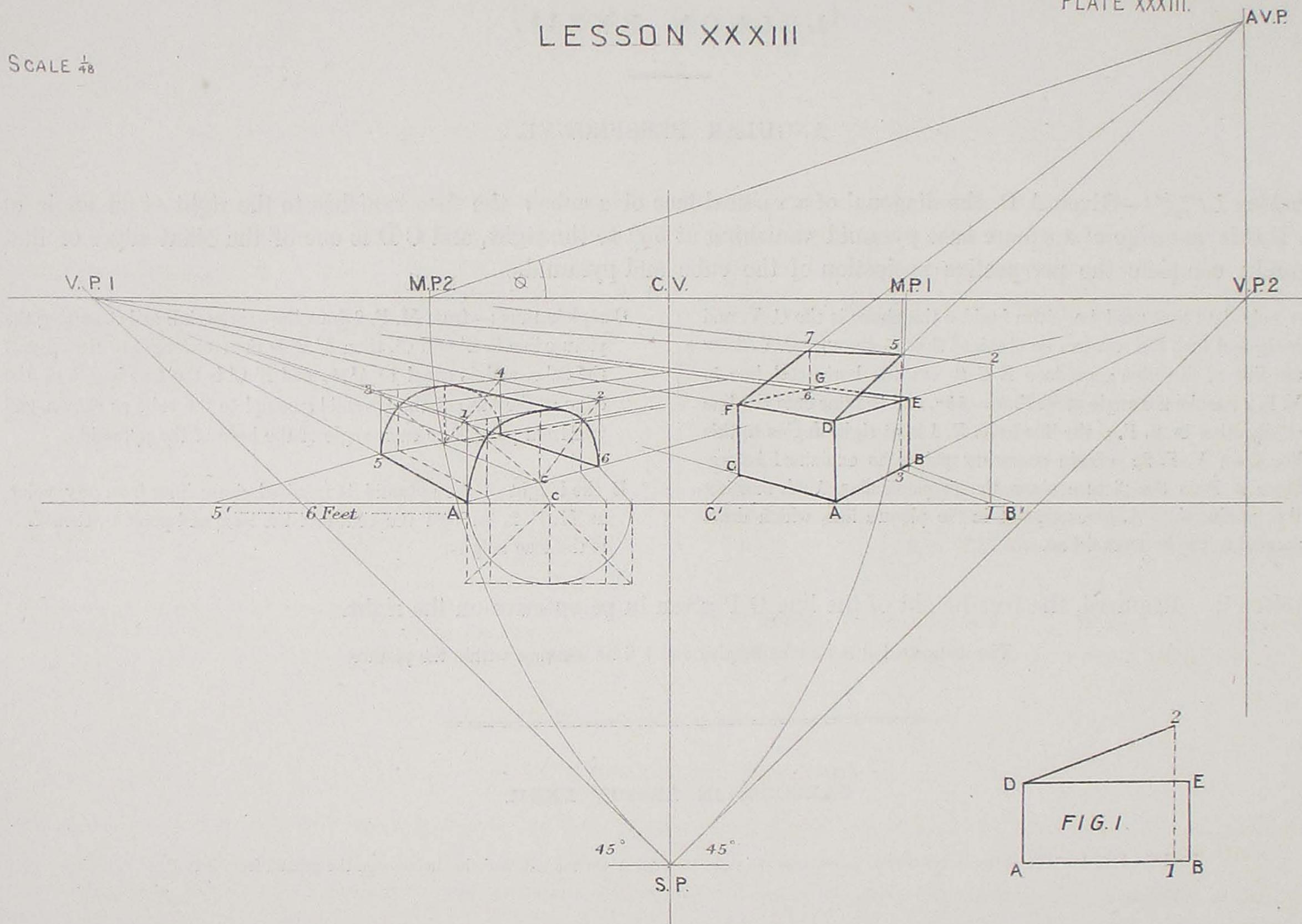
EXERCISE IN LESSON XXXII.

Find the length of the string to reach from point 3 on the right to point A¹ on the left, and the distance on the ground from B to P.

LESSON XXXIII

PLATE XXXIII.

SCALE $\frac{1}{48}$



LESSON XXXIII.

ANGULAR PERSPECTIVE.—(*Distance 14 ft., height of eye 5 ft., scale $\frac{1}{15}$, angles 45° .*)

Problem 1 (on the left.)—A piece of sheet iron, 6 ft. long, is here shown in perspective, bent into a semicircular arch. 4 ft. diameter, and touching the picture plane 5 ft. to the left, its edges making angles of 45° with the picture plane.

This requires very little explanation. Put the rectangle A 1 2 6 in perspective; draw 1 c and 2 c, and from the semicircle below the picture line, obtain points cutting the diagonal 1 c, etc.; complete the curve by hand; obtain similar points in the rectangle 5 3 4, etc., by drawing lines from the former to V. P. 1, and complete as before by hand; finish as shown in the plate.

Problem 2 (on the right.)—Fig. 1 is the end of a square box, with the lid lifted; place it in perspective when point A

touches the picture line 4 ft. to the right, and the side, A B, vanishes to the right at an angle of 45° .

The box, A B C D E F G, is first placed in perspective as explained in (Lesson XXV., etc.)

TO OBTAIN THE COVER.—In Fig. 1, draw the perpendicular line, 1 2; set off on the picture line A 1, equal to A 1 in Fig. 1; draw to M. P. 2, cutting A B in 3; set up from point 1 in the picture line 1.2 = 1.2 (Fig. 1); join to M. P. 2, and draw the perpendicular line 3.5 to cut it in 5; join 5 D; from points 4 and 5 draw lines to V. P. 1, the former cutting F G in 6; set up 6.7, cutting the line from 5 to V. P. 1 in 7; complete as shown with dark lines.

NOTE.—*Accidental Vanishing Points.*—Produce D 5 and F 7: they cut in A V. P. in a line through V. P. 2 perpendicular to the horizontal line: this point is called an *Accidental Vanishing Point*. If a line be drawn from A C P to M. P. 2, the angle marked \odot in the plate will be the same magnitude as the angle 2 D E of Fig. 1, a proof of the correctness of the perspective projection of the box lid.

EXERCISES ON LESSON XXXIII.

- (1) Place the same box in perspective, 4 ft. to the left, and 1 foot within the picture, the side A B vanishing at an angle of 45° to the right, and the lid opened 45° with the edge of the box D E (Fig. 1.)
- (2) Place the piece of iron in perspective (Problem 1) when A is 5 ft. to the right, and 2 ft. within the picture, A C vanishing as before at 45° to the right.

LESSON XXXIV.

SCALE $\frac{1}{48}$

LESSON XXXIV.

ANGULAR PERSPECTIVE.—(*Distance, 14 ft. ; height of eye, 5 ft. ; scale, $\frac{1}{8}$; angles, 40° and 50° .*)

Problem 1 (on the right).—Fig. 1. Out of a solid cube of wood, a piece A B C D G is cut away. Place the remainder A B C D E F in perspective when E touches the picture plane 4 ft. to the right, and E F vanishes to the right at an angle of 50° .

Obtain the V. P.'s, M. P.'s, etc., as explained before. From point E on the picture line, 4 ft. to the right, draw lines to V. P. 1 and V. P. 2, and put the solid cube A G E F K L M in perspective. Draw the dotted lines on Fig. 1 to get the points 1 2 3 4. Mark the points 1 2 on E G¹, and draw to V. P. 2. Obtain the points 3 4 in perspective on E F, and draw to cut the lines going from 1 2 to V. P. 2. The points B and C are thus obtained in perspective. Join A B, B C, C D, and darken as shown. From points 1 D 2 draw lines to V. P. 1, cutting K L in 1¹ D¹ 2¹. Draw lines from the points 1¹ and 2¹ to V. P. 2, meeting lines from B and C to V. P. 1 in B¹ and C¹. Join M B¹, B¹ C¹, C¹ D¹, and darken as shown in the Figure.

NOTE.—If B A and B¹ M are produced they cut in A V. P. 1 ; also B C and B¹ C¹ cut in A. V. P. 2. These points are called Accidental Vanishing Points 1 and 2.

Problem 2 (on the left).—A B C D E F is the perspective projection of a piece of wood, wedge-shaped. Take it out of perspective. The point A is 4 ft. to the left, and above the ground.

Set off on the picture line A¹ 4 ft. to the left, and erect the perpendicular line A¹ 1 any height. A line from the C. V., through A, cuts the perpendicular in 2. A¹ 2 is the height of A above the ground. From A¹ draw a line to C. V., and from A a perpendicular line cutting it in 3. Point 3 is the position of A if it was on the ground. Draw lines from 3 to V. P. 1 and V. P. 2, meeting vertical lines from B and D in B¹ and D¹. The solid block A B B¹ 3 D¹ D C is then represented in perspective. Lines from M. P. 2 through points 3 and B¹ give R K the length of A B. Lines from M. P. 1 through points 3 and D¹ give L M the length of A D. Draw the rectangle A B C D, Figure 2, with the lengths R K and L M. It is the plan of the wedge. The length of B E is found by drawing a line from V. P. 2 through E to E¹, and a line from C. V. through E¹ to point 4 in A¹ 1 ; the distance 2 4 being the length of B E. The triangle 2 4 5, Fig. 2, is the end elevation of the wedge of wood.

NOTE.—A line drawn from P. D., through 3, gives the distance A¹ S that A is within the picture.—(Problem 2.)

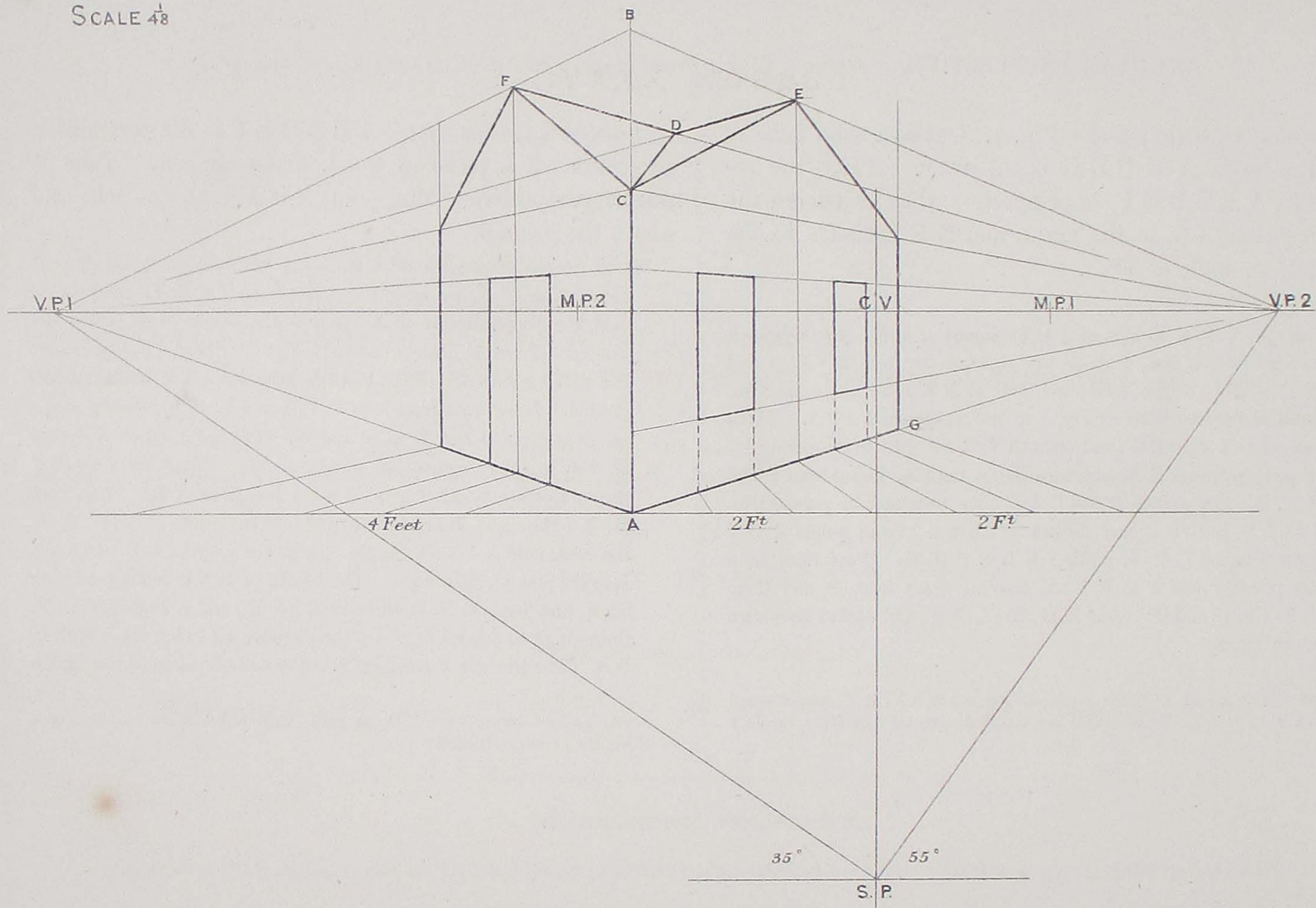
EXERCISE ON LESSON XXXIV.

Place Fig. 1, (Problem 1) in perspective when point E is 2 ft. within the picture, 3 ft. to the right, and E F vanishes at an angle of 50° to the right.

LESSON XXXV.

PLATE XXXV.

SCALE $\frac{1}{48}$



LESSON XXXV.

ANGULAR PERSPECTIVE.

Distance, 14 ft.; height of eye, 5 ft.; scale, $\frac{1}{8}$; angles, 35° and 55° .

Problem 1.—A house 12 ft. square, with two windows on one side and a door on the other, and a gable over the centre of each side, is here shown in perspective, when the corner touches the picture plane, and one side vanishes at 55° to the right.

The method of placing this in perspective will be seen without difficulty. Notice the height of the gable E and F is obtained in C B, and the roofs intersect in D C.

EXERCISE ON LESSON XXXV.

Place the same house in perspective when A is 6 ft. to the right, and 3 ft. within the picture, A G vanishing to the right at an angle of 30° .

LESSON XXXVI

PLATE XXXVI
1877

SCALE $\frac{1}{48}$

LESSON XXXVI.

EXAMINATION PAPER.—SCIENCE AND ART DEPARTMENT, 1877.

PARALLEL AND ANGULAR PERSPECTIVE.

Distance, 14 ft.; height of eye, 5 ft.; scale, $\frac{1}{8}$; angle, 30° ; C. V. given.

Problem 1.—Fig. 1 is the plan of a screen 10 ft. high, A B is at right angles, B C parallel, and C D inclined at 30° to the picture plane. Place it in perspective when A is 10 ft. to the left and 2 ft. within the picture.

Obtain points V. P. 1, V. P. 2, M. P.s, P. D., etc., as before, and find A 10 ft. to the left and 2 ft. within the picture. Set off 3 4=to A B Fig. 1, and draw to P. D. cutting the line from 1 to C. V. in B. Draw B C parallel to the picture line and set off 1 5=to B C, Fig. 1. Join 5 to C. V. cutting the line from B in C. Draw a line from V. P. 1 through C, and a line from M. P. 1 through C. Set off R 6=C D Fig. 1, and from 6 draw to M. P. 1 cutting C D in D. Set up the height 1-2 10 ft. high, and complete as shown.

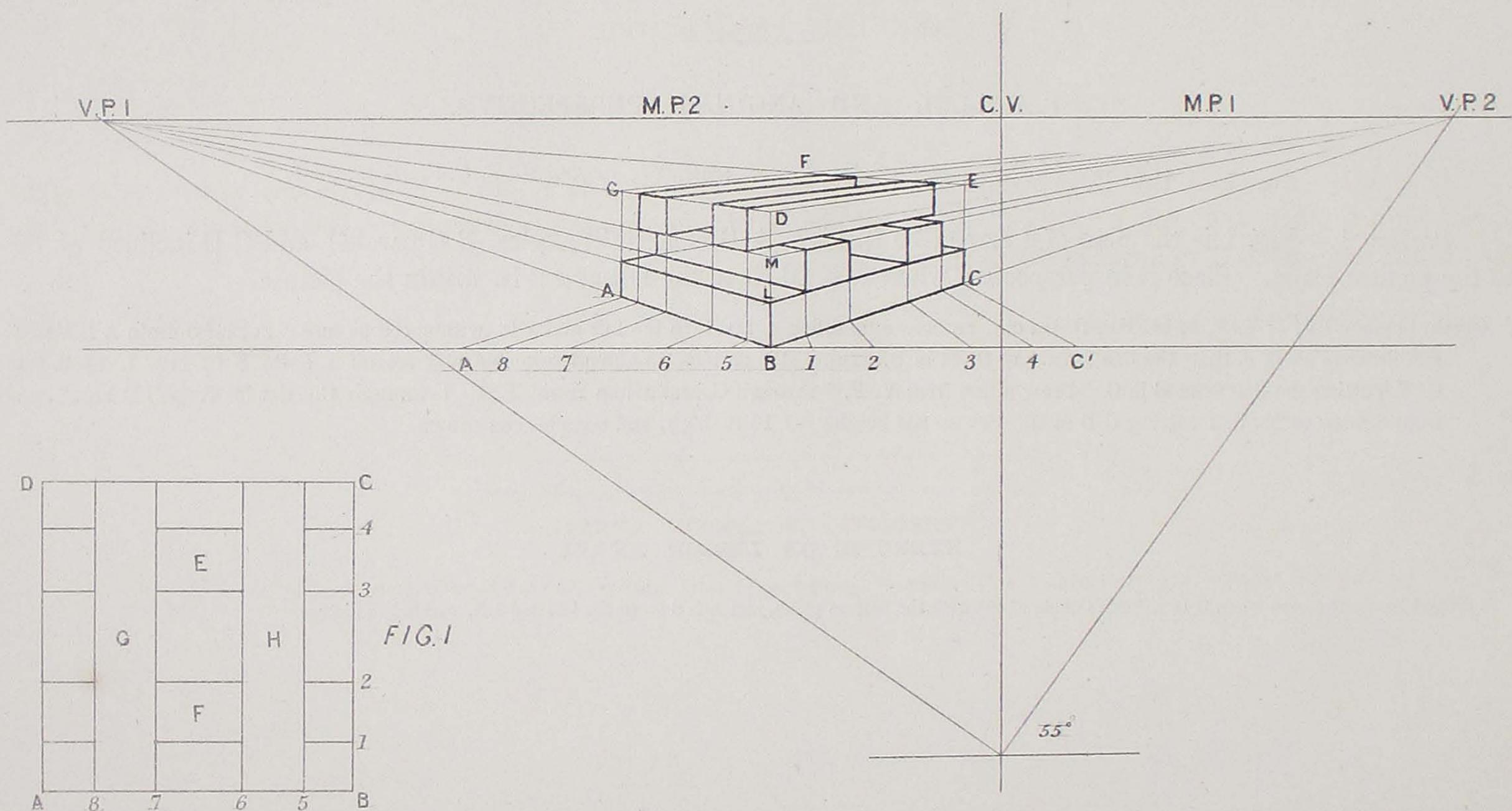
EXERCISE ON LESSON XXXVI.

Project the same screen when C D makes an angle of 40° with the picture plane, and A is 6 ft. to the left and 3 ft. within the picture.

LESSON XXXVII.

PLATE XXXVII.

SCALE $\frac{1}{48}$



LESSON XXXVII.

ANGULAR PERSPECTIVE.

Distance, 14 ft. ; height of eye, 5 ft. ; scale, $\frac{1}{8}$; angles, 35° and 55° .

Problem 1.—Fig. 1, A B C D, is a solid block of wood 1 ft. thick; E F and G H are blocks laid on it at right angles to each other, and each 1 ft. thick. Place all the blocks in perspective when point B touches the picture plane 5 ft. to the left, and B C vanishes at 55° to the right.

Obtain V. P.'s, M. P.'s, etc., as before explained. Put the solid block A B C D E F G, 3 ft. thick, in perspective with B, 5 ft. to the left. Set up B L and L M each 1 ft. Draw from L M to V. P. 1 and V. P. 2. On the picture line mark the points B 1 2 3 4 C¹ and B 5 6 7 8 A¹ corresponding to similar points in Fig. 1. Draw to the respective M. P.'s cutting B A and B C. Complete as in the drawing.

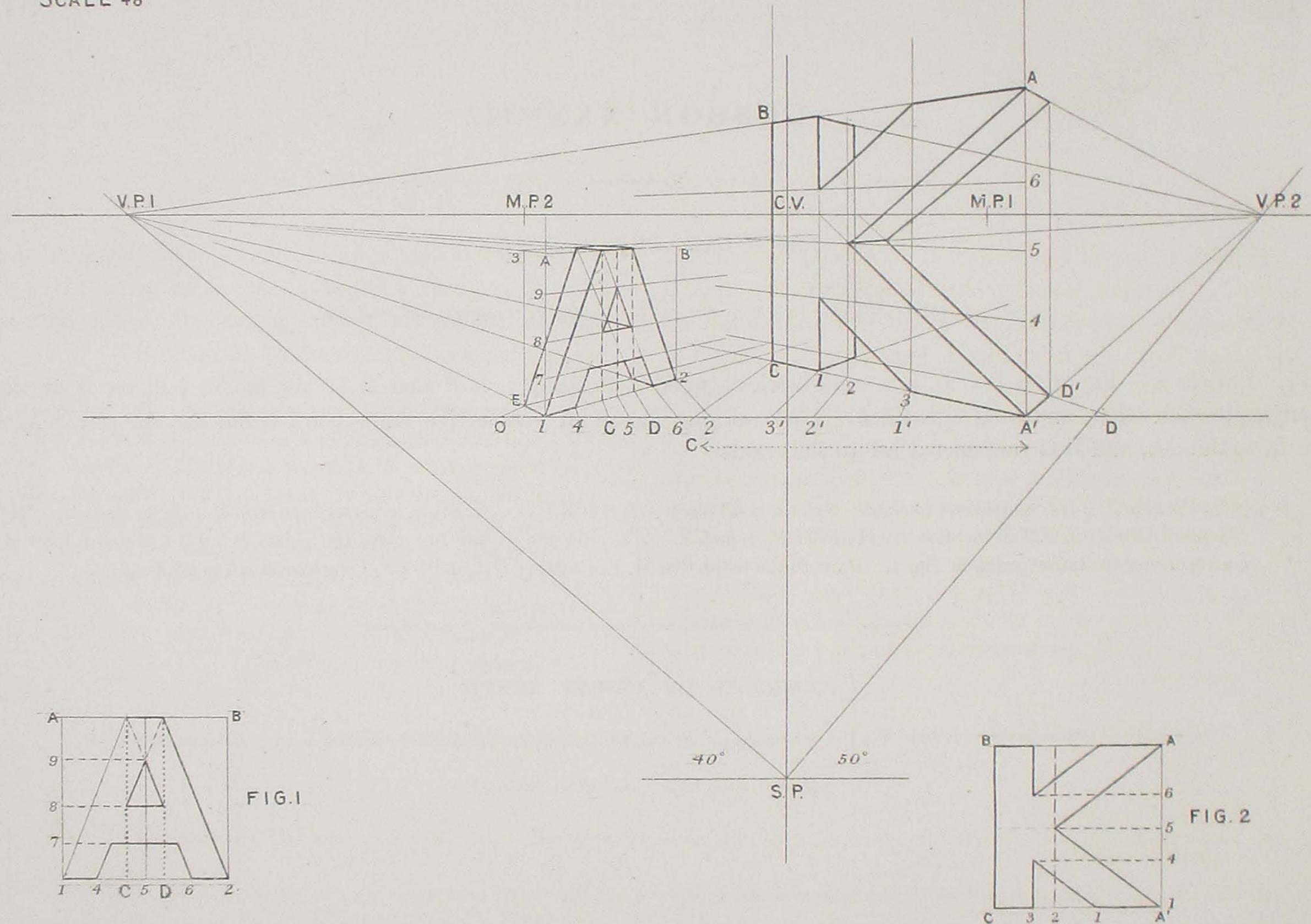
EXERCISES ON LESSON XXXVII.

Place the same blocks in perspective when B is 2 ft. within the picture and directly opposite the eye, B C vanishing to the right at an angle of 45° .

LESSON XXXVIII.

PLATE XXXVIII.

SCALE 48



LESSON XXXVIII.

ANGULAR PERSPECTIVE.

Distance, 14 ft.; height of eye, 5 ft.; scale, $\frac{1}{8}$; angles, 50° and 40° .

Problem 1 (on the left).—Fig. 1. The elevation of the letter A, 1 ft. thick, is given, place it in perspective when it touches the picture plane 6 ft. to the left and stands on the ground with the face, Fig. 1, vanishing to the right at an angle of 50° .

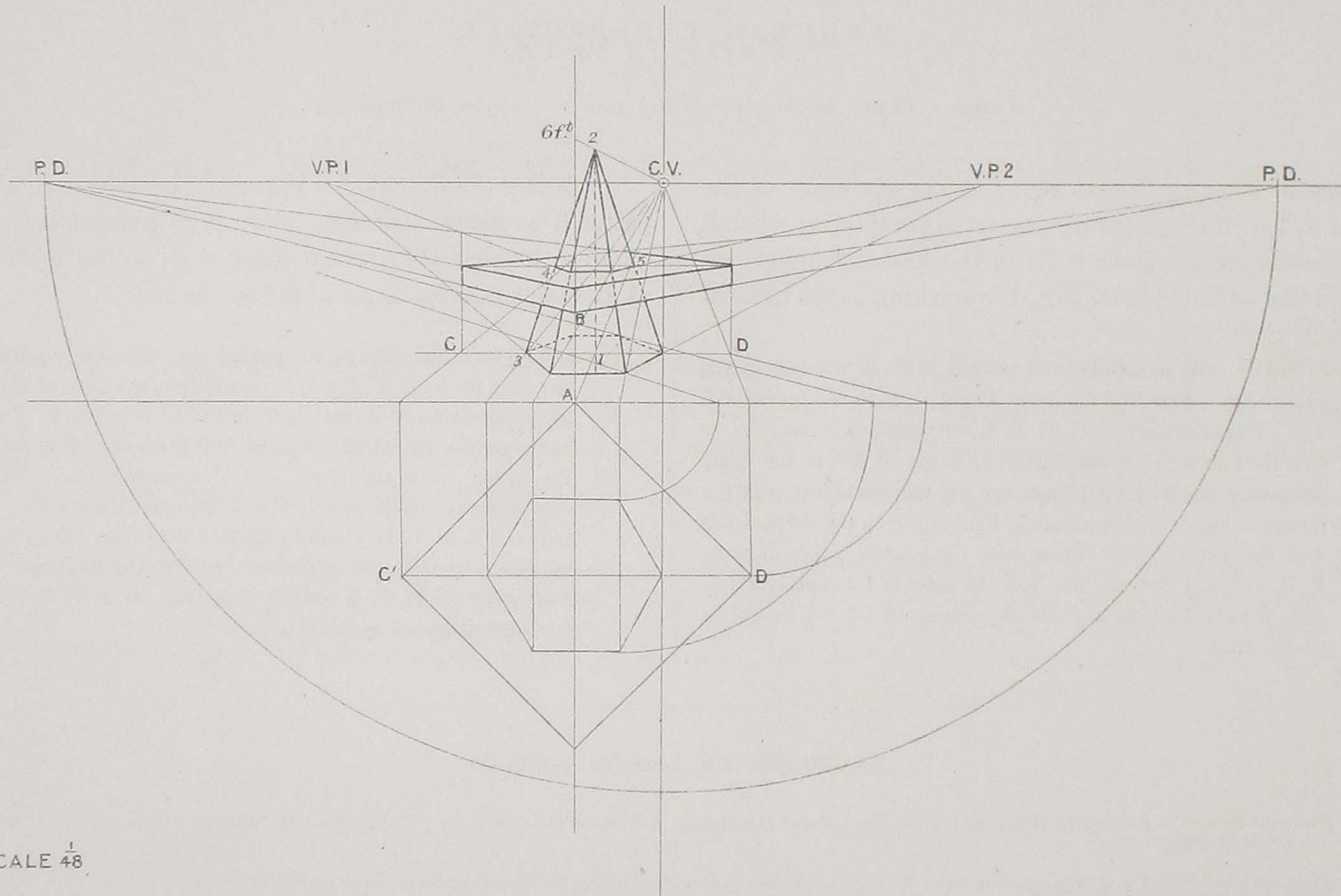
Obtain V. P.'s, etc., as before, and point 1, 6 ft. to the left in the picture line. Surround the letter A, Fig. 1, with the square A B 1 2. Place the block, A B, 21, E, 3, 1 ft. thick, in perspective, so that the face A B 1 2 vanishes at an angle of 50° to the right. Between 1 and 2 on the picture line get the points 4 C, 5 D, 6 2 corresponding to similar points in Fig. 1, and on the vertical line 1 A the points 7 8 9. Draw from these latter points lines to V. P. 2 cutting vertical lines from the point in 1 2 in perspective, and complete the letter as shown, by setting off 1-0 1 ft. the thickness of letter.

Problem 2 (on the right).—Fig. 2. The elevation of the letter K is given, half size, place it in perspective when point A touches the picture plane 6 ft. to the right and A B vanishes at an angle of 40° to the left.

Set off A¹ 6 ft. to the right on the picture line. Put the square surrounding the letter K, Fig. 2, in perspective, as shown at A¹ A B C, taking care that A¹ A, etc., is double of A¹ A in Fig. 2. In A¹ C on the picture line set off the points 1¹ 2¹ 3¹ corresponding to 1 2 3, Fig. 2, but double the length. In A¹ A mark points 4 5 6 corresponding to similar ones in Fig. 2, also double the size. Draw from 4 5 6 to V. P. 1 cutting vertical lines from 1 2 3; darken the face of the letter K as shown. Set off the thickness (2 ft.) A¹ D, draw to M. P. 2 cutting lines from A¹ to V. P. 2 in D¹. Complete as shown in the figure.

EXERCISES ON LESSON XXXVIII.

- (1) Place the letter A in perspective (Problem 1) when Fig. 1 shows it half size. Let the point 1 touch the picture plane 6 ft. to the right, and A B vanish to the right at an angle of 50° .
- (2) Place the letter K in Fig. 2 in perspective under the same conditions as above in Problem 2, but point A to be also 2 ft. within the picture.

SCALE $\frac{1}{48}$

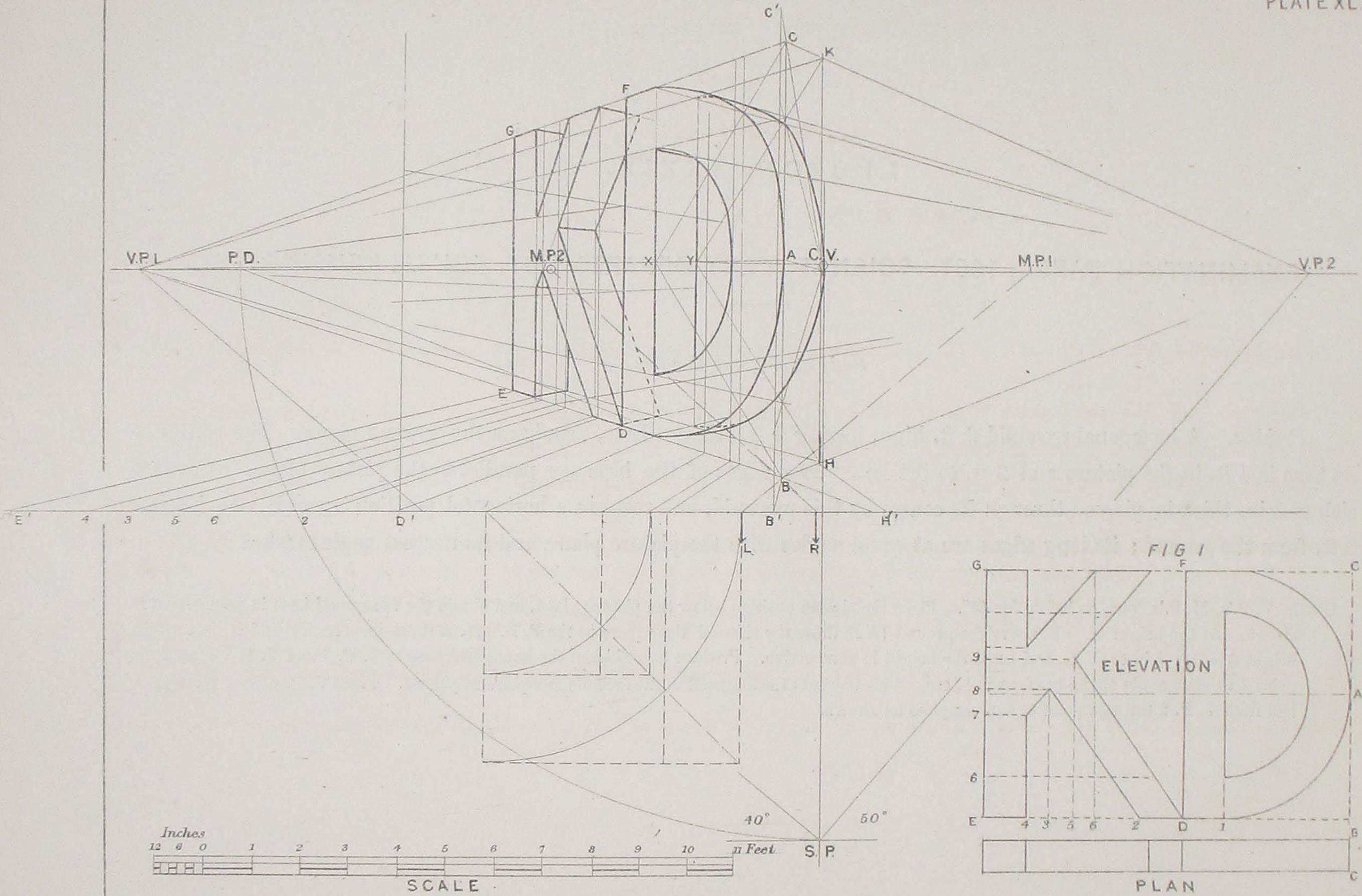
LESSON XXXIX.

EXAMINATION PAPER, 1867.—SCIENCE & ART DEPARTMENT, SOUTH KENSINGTON.

Scale $\frac{1}{8}$, or $\frac{1}{4}$ inch to a foot.

Problem.—A hexagonal pyramid 6 ft. high, having 2 ft. sides of base, stands on the ground plane. The centre of its base is 4 ft. in the picture and 2 ft. to the left. Two edges of the base are parallel to the picture plane. A square slab of 6 in. thick is pierced through its centre by this pyramid, and rests in a horizontal position upon it, at a level of 2 ft. from the ground; its long edges are at equal angles with the picture plane, and its nearest angle touches it.

Obtain V. P.'s, M. P.'s, etc., as before for 45° . Place the plan in position below the picture plane, and obtain the hexagonal base in perspective as shown. At the height of 2 ft. above the ground (A B) draw the sides of the square to the P. D.'s; cut these lines from C and D; set up the height A 6 ft.; join to C. V., and 1 2 is the height in perspective. Produce two sides of the hexagonal base to V. P. 1 and V. P. 2; join from point 2 to each angle of the base, as 2, 3, etc. This line cuts the diagonal of the face of the square in point 4. From V. P. 1 draw through 4, and from V. P. 2 through point 5, and complete as shown.



LESSON XL.

EXAMINATION PAPER.

MAY, 1876, AND MARCH, 1879.

DIRECTIONS.—The C. V. is given. The eye of spectator is to be 12 ft., by scale, distant from it, and 5 ft. above the ground plane.

Problem.—Give the perspective representation of the two letters, shown half size in the plan and elevation Fig. 1. The faces of the letters lie in planes receding from the picture plane at an angle of 40° towards the left. The point A in the letter D is to be 1 ft. on the left of the spectator, and 2 ft. within the picture.

Having obtained the V. P. for 40° on the left, etc., find a point B 1 ft. to the left and 2 ft. within the picture. Place the solid block B E G C H K in perspective, all measurements being twice the size as given by the plan and elevation, and the thickness twice B C (Fig. 1 Plan). Set off on the picture line the points corresponding to those on B E (Fig. 1); join to M. P. 1; cutting B E; vanishing to V. P. 1 in corresponding points. On B¹ C¹ set up points 6, 7, 8, 9, and obtain them on B C by joining to C. V. Draw from these points to V. P. 1, cutting vertical lines from 1, 2, 3, etc.

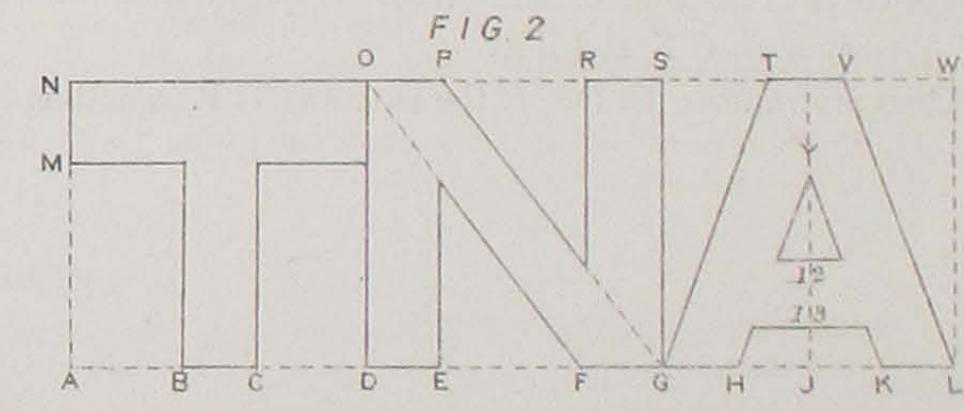
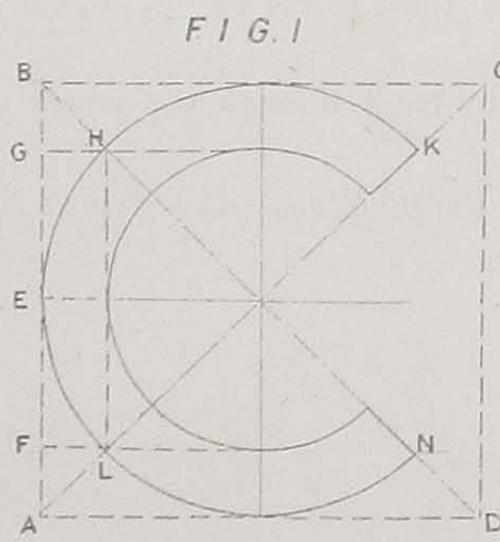
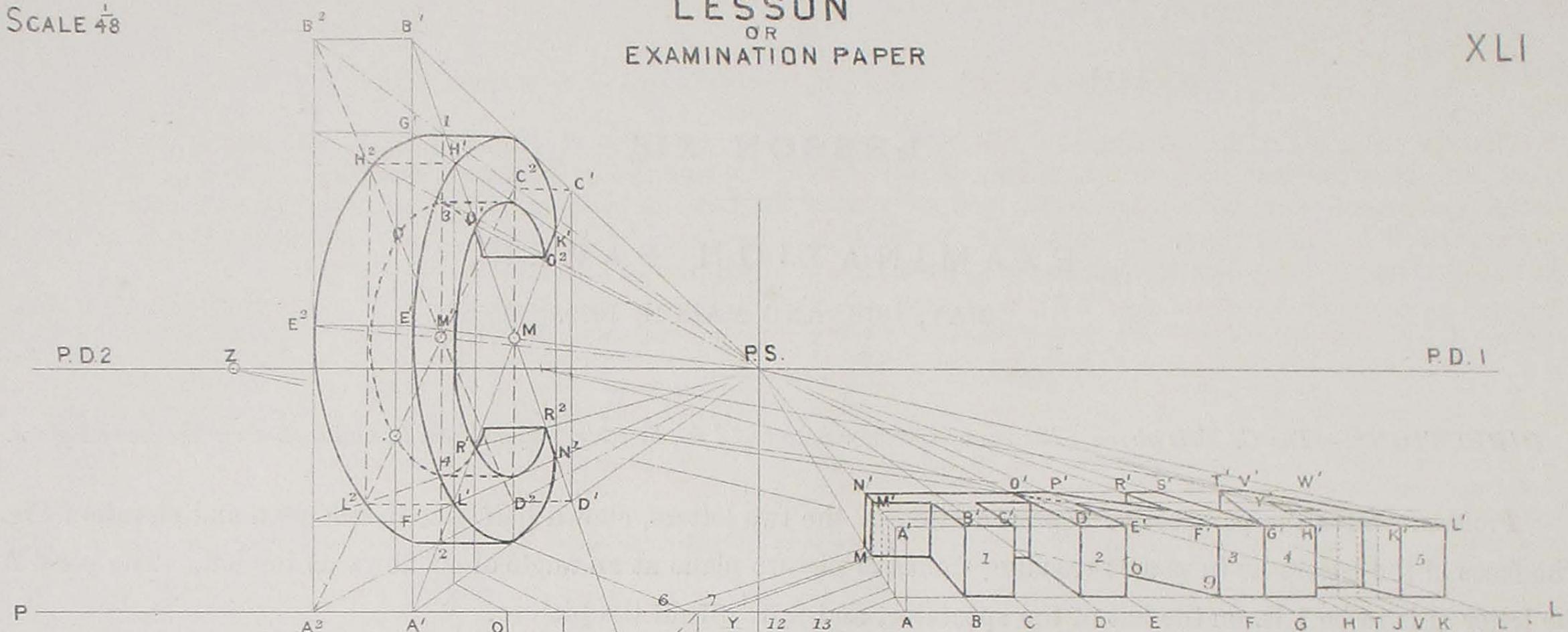
The curve of the D is obtained from the quadrant shown below the picture line, drawn in perspective by hand through the points cutting the diagonals C X and B X.

Lines drawn from the face of the letters to V. P. 2, cutting lines from H and K, etc., to V. P. 1, will show the thickness. Complete as in the figure.

SCALE 1/8

LESSON
OR
EXAMINATION PAPER

XLI



LESSON XLI.

EXAMINATION PAPER.—*Scale, etc., as in previous Lesson.*

On the left, Fig. 1; is the elevation of the block letter C (half the required size). It is 2 ft. thick. Place it in parallel perspective at right angles to the ground plane, and the point E touching the picture plane 7 ft. to the left. The face of the letter to be at right angles to the picture plane.

Surround the letter with the dotted lines A B C D. Set off on the P. L. $A^1 7$ ft. to the left, and draw the vertical line $A^1 B^1 =$ twice A B (since A B is half size). Join B^1 to S. P., as explained in Lesson VII., and complete the square $A^1 B^1 C^1 D^1$ in perspective. Draw the diagonals cutting in M. In $A^1 B^1$ set off the points $F^1 G^1$ corresponding to F G, Fig. 1. Join $F^1 G^1$ to P. S. cutting the diagonals in $H^1 K^1 L^1 N^1$. Draw the oval by hand to pass through these points and touch the square $A^1 B^1 C^1 D^1$. Darken the outer circumference, as shown, terminating at $K^1 N^1$ by the diagonals, to correspond to Fig. 1. The inner circumference in the face, $A^1 B^1 C^1 D^1$, is enclosed within the square $H^1 L^1 K^1 N^1$ (see H K L N, Fig. 1). All that is wanted to complete this part is to get the points where this small circle, or rather part of circle, cuts the diagonals $H^1 N^1$ and $L^1 K^1$. This is easily obtained from the quadrant O R P described in the position shown below P. L. Draw lines to P. D.² and then vertically cut the diagonal $H^1 N^1$ in O¹ and $L^1 K^1$ in R¹. A line from O¹ R¹ to P. S. gives the other two points O² R². Complete as shown.

The letter is 2 ft. thick.—Set off $A^1 A^2 = 2$ ft. by scale, and complete the square $A^2 B^2 C^2 D^2$. Draw its diagonals $B^2 D^2$, $A^2 C^2$ cutting in M¹. Draw the horizontal lines $L^1 L^2$, $H^1 H^2$ cutting these diagonals in $H^2 L^2$. Follow the vertical line through M¹ till it meets the lines from B² A² to P. S. (points 1 2.) Describe the semi-oval 1 H² E² L² 2, and draw the horizontal line from 1 2 to meet the front oval. As in the front face, $H^1 L^1 K^1 N^1$ enclosed the smaller circle, so now H² L² 3 4 encloses the corresponding half-oval. Darken in what is seen, and complete as in the figure.

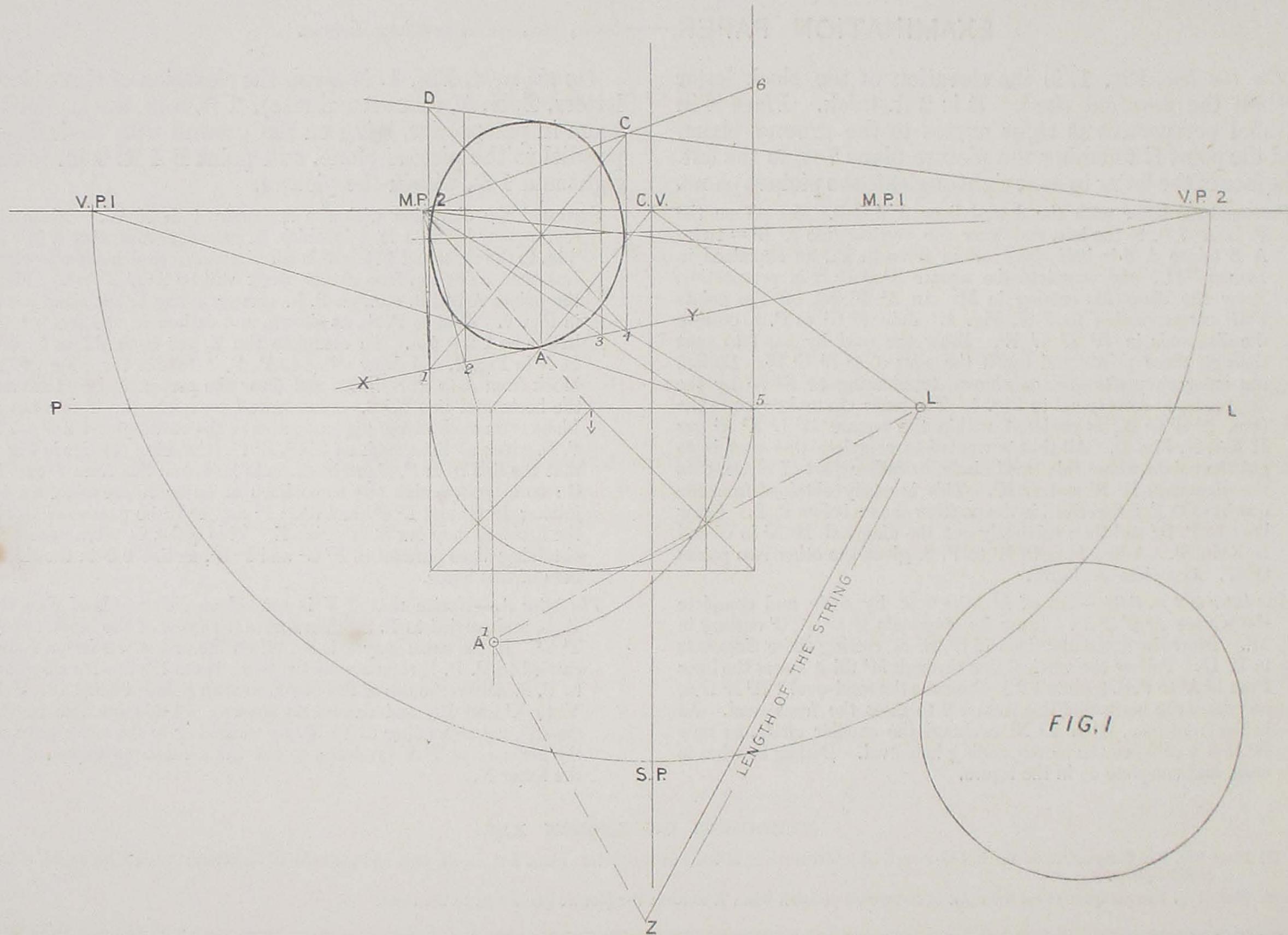
On the right, Fig. 1, is given the elevation of three block letters, T, N, A, (shown full size), 1 ft. 6 in. thick. Place them in perspective, lying on the ground with their bases parallel to the picture plane, and point B 4 ft. 6 in. to the right and 1 ft. within the picture.

Surround the elevation with the rectangle A L N W. Place the rectangular block, 1 ft. 6 in. thick, in perspective, so that B is 4 ft. 6 in. to the right and 1 ft. within the picture, as explained in previous Lessons. (The top face of this block will be $A^1 L^1 N^1 W^1$.) Mark the points A B C D, etc., on P. L., corresponding to the same points in Fig. 2. Join to P. S., as shown, and darken in the feet of the letters as 1 2 3 4 5. To complete the T, set back 6.7 in P. L. = M N in Fig. 2, and draw a line to P. D. 1 to cut the edge of the block from A to P. S. in M, and draw the perpendicular M M¹ and the horizontal line M¹ 8. Darken and complete the T as shown. Lines from each of the upper corners of the rectangles 1 2 3 4 5 to P. S. gives all lines such as D¹ O¹, E¹ P¹, G¹ S, etc. Notice in Fig. 2 that the line from P, if produced, cuts in G, and line from F cuts in O; so in perspective the same lines of letter N are obtained by joining P¹ G¹ and F¹ O¹, and if G¹ P¹ and F¹ O¹ be produced to cut the horizon, they cut it in point Z. This point is often useful in obtaining lines parallel to P¹ G¹ and F¹ O¹, as line 9.0 in the thickness of this letter.

The letter A.—Notice that T V is set off on either side of J on the P. L. and carried to P. S., then across the faces of the block to join T¹ V¹. In the same way Y is set off on the end of the letter T, and carried to P. D. 1, then across the block to cut the line from J going to P. S. across the top of the block, so with points 12 and 13. Join Y¹ to K¹ and H¹, and darken as shown. Obtain the line on the ground, corresponding to Y¹ K¹, by vanishing to the same point in the horizon, as Y¹ K produced to cut the horizon, as explained for the letter N.

EXERCISES ON LESSON XLI.

- (1) Place Fig. 2 in perspective on the left in a vertical position when M touches the picture plane 8 ft. to the left, and the faces of the letters are at right angles to the picture plane.
- (2) Place Fig. 1 in perspective on the right in horizontal position when E touches the picture plane 6 ft. to the right.



LESSON XLII.

EXAMINATION PAPER.—(H. M. S. "BRITANNIA")

FOR NAVAL CADETS AND MIDSHIPMEN.—*Scale $\frac{1}{48}$ or $\frac{1}{4}$ in. to a foot.*

Problem 1.—A circle of 8 ft. diameter stands on the ground plane, and is on a plane vanishing to the right at an angle of 45° . Place it in perspective when its centre is 4 ft. to the left and 6 ft. within the picture plane. Distance 14 ft., height of eye 5 ft.

Obtain point A 4 ft. to the left, and 6 ft. within the picture plane. Draw from V. P. 2, through A, the line X Y, and a line from M. P. 2, through A, to P. L. Describe the semicircle, 4 ft. radius, below the picture line where shown, and enclose it as explained before, drawing lines to M. P. 2, cutting X Y in 1 2 3 4. Set up 5 6, the diameter of the circle, and complete the square in perspective, 1 4 C D. Draw the diagonals, and complete the circle by hand.

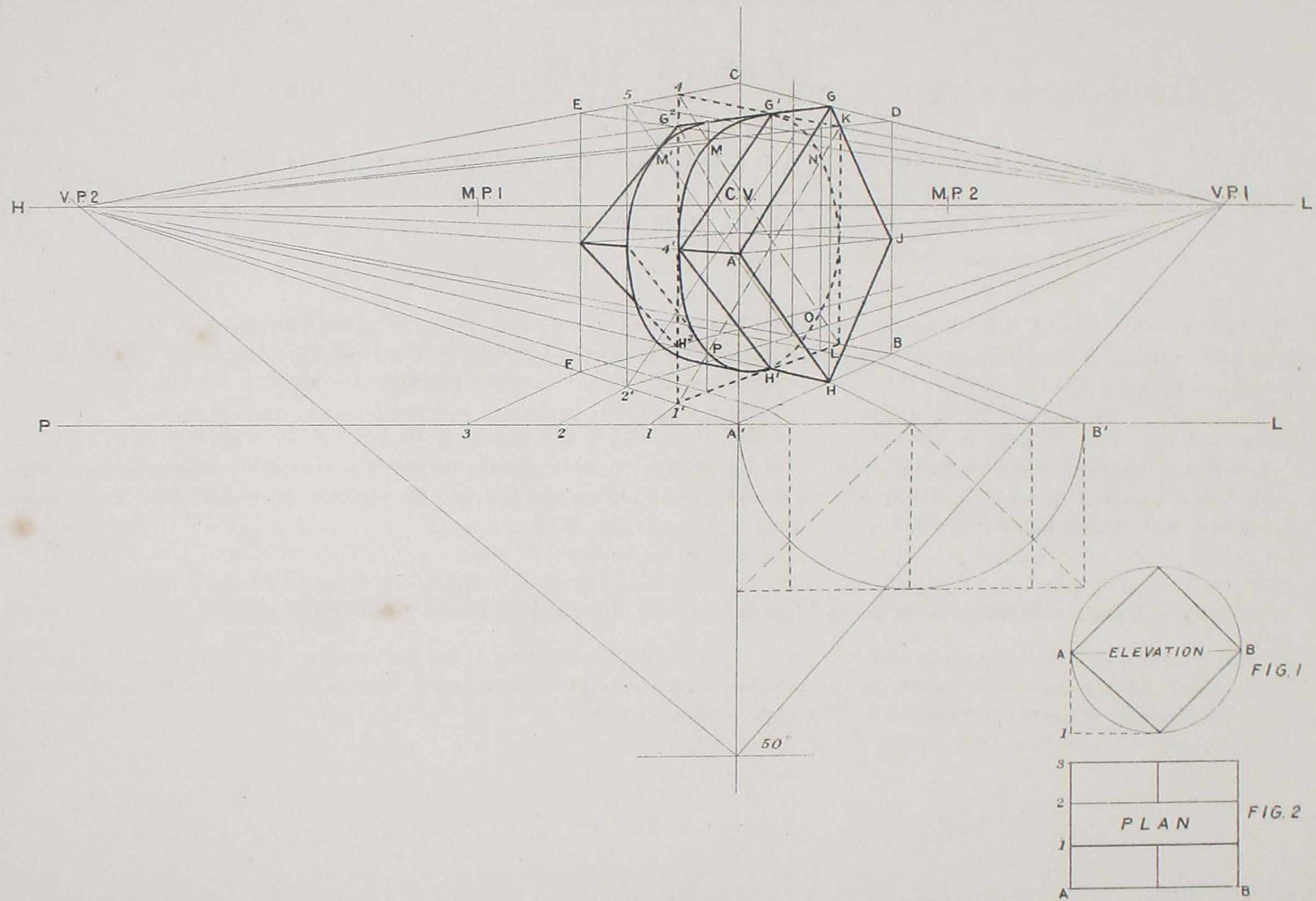
Problem 2.—Find the distance from point L on the picture line to the highest point of the circle.

Take point A out of perspective (A^1) by lines from P. D. and C. V. as explained before, then $A^1 L$ is the base of a right-angled triangle. Make the height ($A Z$) 8 ft. the same as the diameter of the circle, because the highest point must be 8 ft. above the ground. $L Z$, the hypotenuse of the triangle, is the length of the string that will reach between the two points.

EXAMINATION PAPER—SCIENCE & ART DEPARTMENT.

SCALE 48

XLIII



LESSON XLIII.

EXAMINATION PAPER.—SCIENCE AND ART DEPARTMENT, MAY, 1876.

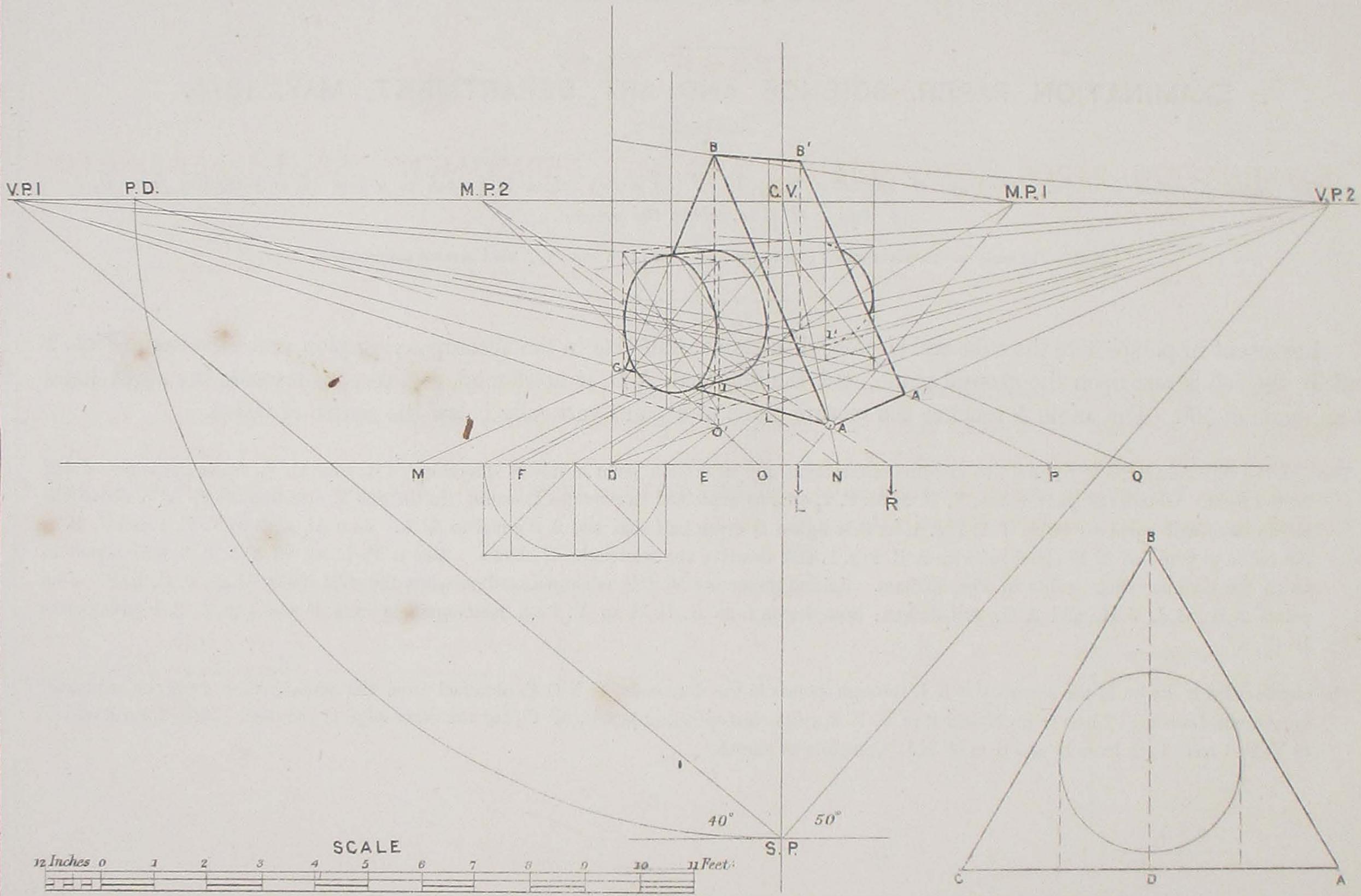
The centre of vision is given. The eye of spectator to be 14 ft. (by scale) which is $\frac{1}{4}$ inch to a foot ($\frac{1}{48}$ in.) distant from it, and 5 ft. 6 in. above the ground.

N.B.—The scale on the Government Papers was ($\frac{1}{44}$) $\frac{1}{2}$ inch to a foot; ($\frac{1}{48}$) used here for convenience.

Represent in perspective the cube and circular disc shown half size in the accompanying plan and elevation, (Figs. 1 and 2), the solids rest upon the ground plane, and the diagonal, A B, is horizontal, and recedes towards the right hand at an angle of 50° . The angle A touches the picture plane at a point vertically below the centre of vision.

Through the C. V. draw the horizontal line and line of direction, and at station point in line of direction, 14 ft. from C. V., make the angle of 50° on the right. Obtain the picture line, V. P. 2, M. P. 2, etc., as explained in previous Lessons. In the line of direction set up A¹ A above the picture line, and equal to twice 1 A, Fig. 1, as this figure is given half size, and A C equal to A¹ A. Join A¹, A, C to V. P. 1 and V. P. 2. Set off on picture line A¹ B¹ equal to twice A B, Fig. 1, and describe the semicircle as shown. Also in P. L. set off A¹ 1, 2, 3, each equal to twice the corresponding points in Fig. 2 (Plan). Join to respective M. P.'s as shown, and complete the solid block A¹, B, D, C, E, F. Join points A G, G J, J H, and A H, and darken; lines drawn from G, H, A to V. P. 2, meeting lines from 1¹ and 4 to V. P. 1 give points H¹ G¹, 4¹.

The circular disc is drawn in the square 1¹ 4 K L through points in the diagonals M N O P, obtained from the semicircle on A¹ B¹, as explained in previous Lesson. Lines from M and P to V. P. 2 gives corresponding points, M¹ P¹, for the other edge of the disc. Lines from E and F to V. P. 1 meet lines from H and G in G² H². Complete as shown.



LESSON XLIV.

EXAMINATION PAPER, 1879.—SCIENCE AND ART DEPARTMENT, SOUTH KENSINGTON.

Centre of vision given; eye of spectator 12 ft. by scale distant from it, and 5 ft. above the ground.

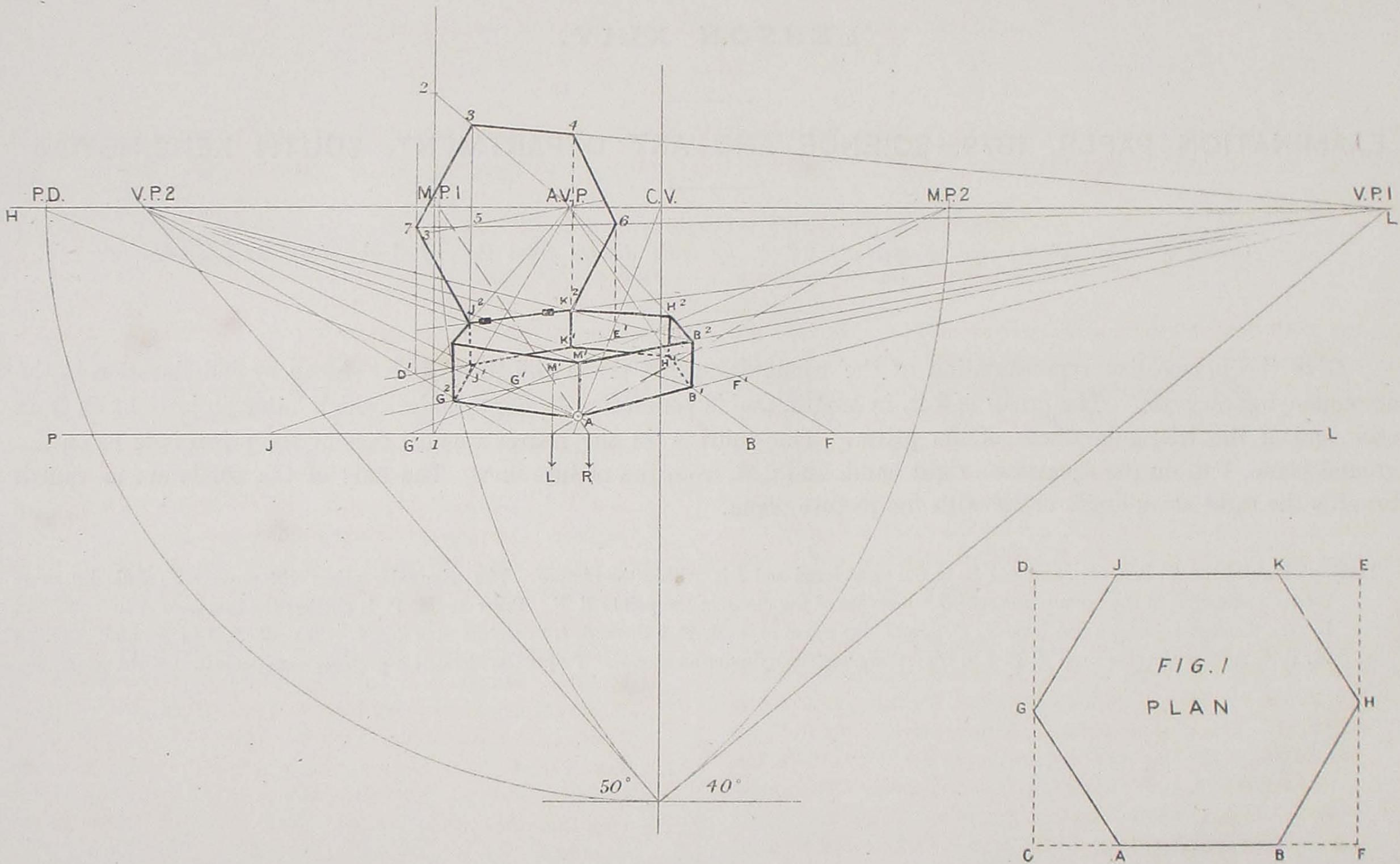
Give the perspective representation of the triangular right prism and the cylinder shown by end elevation in the accompanying diagram. The prism is 3 ft. in length, and is penetrated by the cylinder, which latter projects $1\frac{1}{2}$ ft. from each end of the triangular faces of the prism. The point A in the nearer vertical face of the prism is to be on the ground plane, 1 ft. on the spectator's right hand, and 2 ft. from the picture line. The axis of the solids are to vanish towards the right at an angle of 50° with the picture plane.

Having obtained the V. P.'s, etc., find A 1 ft. to the right hand and 2 ft. within the picture. The triangular prism will present no difficulty now. On each side of D in the picture line set off the radius of the circle in the points E, F. Draw to M. P. 1, cutting the line from A to V. P. 1 in L, etc., through L bring a line from V. P. 2, and also a line from M. P. 2 through L to picture line in N. Set off N O $1\frac{1}{2}$ ft., and draw to M. P. 2, cutting the line from V. P. 2 in O¹. Complete the cylinder as shown. P Q is the distance it projects from back face of the prism, etc.

TRAINING COLLEGE
EXAMINATION PAPER NOV. 1879
SCIENCE & ART DEPARTMENT - SOUTH KENSINGTON.

SCALE $\frac{1}{48}$

XLV



LESSON XLV.

TRAINING COLLEGE EXAMINATION PAPER, NOVEMBER, 1879.

(SCIENCE AND ART DEPARTMENT, SOUTH KENSINGTON.

Fig. 1 is the plan of a tin box, $1\frac{1}{2}$ ft. deep, with lid open at right angles. Place it in perspective when the point A is 2 ft. to the left of the spectator, and 1 ft. within the picture, and the side, A B, vanishes towards the right at an angle of 40° .

(*Height of the eye, 5 ft. 6 in. Distance, 14 ft. 6 in. Scale, $\frac{1}{48}$.*)

N.B.—On the Examination Paper the scale was $\frac{1}{48}$, or $\frac{1}{2}$ inch to a foot, altered here to suit the Plate.

Draw the horizontal line, P. L., etc., and obtain V. P.'s, M. P.'s, etc., as explained in the foregoing Lessons. Find the point A 2 ft. to the left, and 1 ft. within the picture. Draw A B¹ to V. P. 1. Set off on P. L. R B equal to A B (Fig. 1.) Draw from B to M. P. 1, cutting the line from A in B¹. A B¹ is the side A B (Fig. 1) in perspective. Draw the line from A to V. P. 2, and set off on the P. L. L J equal to A J (Fig. 1.) Draw from J to M. P. 2, cutting the line from A to V. P. 2 in J¹. A line drawn from J¹ to V. P. 1, and from B¹ to V. P. 2 cuts in K¹; J¹ K¹ is the side J K (Fig. 1) in perspective. On the P. L. set off L G¹ equal to C G (Fig. 1.) Join from G¹ to M. P. 2, cutting the line from A to V. P. 2 in G¹. Through this point, G¹, draw a line to V. P. 1. Set off B F in the P. L. equal to B F (Fig. 1.) Draw to M. P. 1, cutting the line from A to V. P. 1 in F¹. From F¹ draw a line to V. P. 2, cutting the line from G¹ in H¹. This is point H (Fig. 1) in perspective. Join B¹ H¹: it is the side, B H, (Fig. 1) in perspective, produce it to cut the horizontal line in A. V. P. (Accidental Vanishing Point.) The parallel side G J (Fig. 1) will also meet on this point. A line drawn from A. V. P. through J¹, cutting the line from V. P. 1 through G¹ in G²,

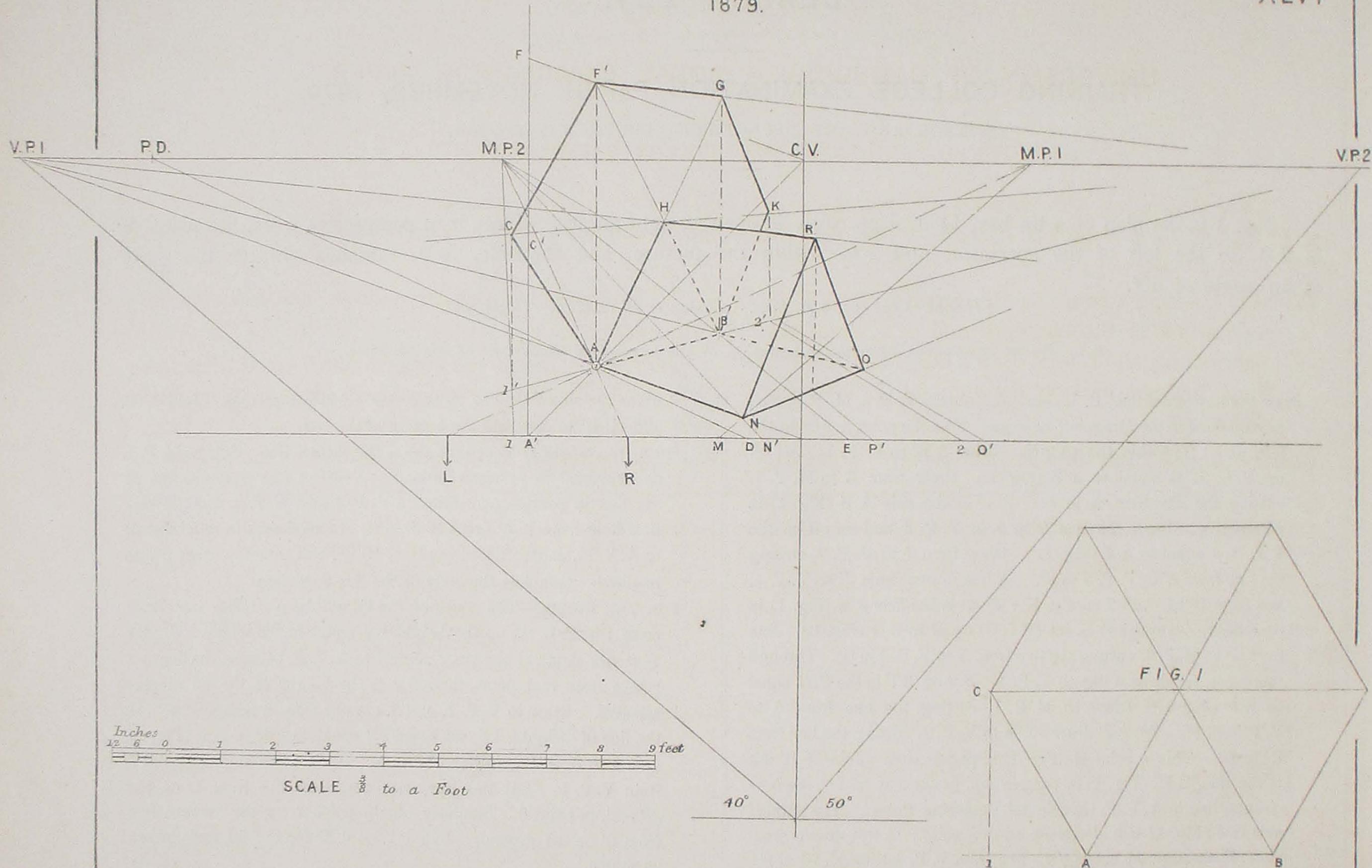
gives the side J G (Fig 1), that is J¹ G² in perspective. Join A G² and H¹ K¹ to complete the bottom of the box.

Set up the height L M $1\frac{1}{2}$ feet above the picture line. Join to M. P. 2 as explained in previous Lessons. A M¹ is the vertical edge of the box in perspective: draw to V. P. 1 and V. P. 2, and the vertical lines from B¹ J¹ gives B² J², etc. Lines drawn from B² and J² to A. V. P., and vertical lines from H¹ G², will give the other points required. Complete the body of the box as shown.

FOR THE COVER.—The width of the lid will be A J (Fig. 1). From point 1 in P. L. set up the height 1.2, equal to the width A J, plus $1\frac{1}{2}$ ft. (the depth of the box). Join 2 to A. V. P. because the line to 1 comes from that point, then J² 3 is the height of the lid in perspective. Draw to V. P. 1, and 3.4 is one side in perspective. In the line of heights, 1.2, set down 2.3 equal to half of A J (Fig. 1). Join 3 to A. V. P., cutting J² 3 in 5. Draw a line through point 5 from V. P. 1. Cut this line, with a vertical line from E¹ on the ground, in point 6. Similarly obtain point 7 by the vertical line from D¹ on the ground. Join J² 7.3.4 6 K² for the lid, and darken as shown.

LESSON
CAMBRIDGE LOCAL EXAMINATION.
1879.

XLVI



LESSON XLVI.

UNIVERSITY OF CAMBRIDGE.—LOCAL EXAMINATION PAPER.

JUNIOR STUDENTS, 1879.—LINEAR PERSPECTIVE.

These Problems are to be worked to a scale of $\frac{3}{8}$ inch to a foot, the distance of the spectator from the picture being in each case 12 ft., and the horizon 5 ft. above the ground plane. The lines by which each Problem is solved must appear on the Paper sent up to the Examiners.

Question 1.—Find a point on the ground plane, 5 ft. on the left of the spectator and 4 ft. from the picture line, and from it draw a horizontal line 4 ft. long, inclined towards the right at 50° with the picture.

Question 2.—Let this line be one side of a regular

hexagon in a vertical position, and complete the figure.

Question 3.—Divide the hexagon into 6 equal triangles, and take that one which has one side on the ground plane as the end of a regular prism, 5 ft. long, projecting from the hexagon towards the picture.

(1) Obtain V. P. 1, V. P. 2, M. P.'s, &c., as explained before—*using the scale $\frac{3}{8}$ " to a foot.*

Set off on the picture line A' 5 ft. to the left of the line of direction, and join to the C. V. Set back from A' on the picture line A'D 4 ft., and join D to P. D., cutting the line from A' to C. V. in A. A is then 5 ft. to the left and 4 ft. within the picture, as required in Question 1.

Draw from A to V. P. 2 and from M. P. 2 a line through A to the picture line (shown by R). Measure off R E 4 ft., the length of the line in Question 1, and join from E to M. P. 2, cutting the line from A to V. P. 2 in B. A B is the line required by Question 1.

(2) In a corner of the Paper construct the hexagon (Fig. 1), having A B, one side, 4 ft. long. Let fall C I in this Fig. perpendicular to A B. At A' in the picture line erect the perpendicular line A'C'F equal to twice the height I C in Fig. 1. Join F to C. V, cutting it by the perpendicular line from the point A in F'. Join F' to V. P. 2, and cut it by a perpendicular line from point B in G. F' G is a side of the hexagon. Join A G and B F', cutting in H. This is the centre of the

hexagon. A line from V. P. 2 through H will contain the two other points (C and K) of the hexagon. Obtain them thus: Set off on the picture line E 2 and R 1 distances equal to A 1 in Fig. 1, and join to M. P. 2, cutting the line through A B to V. P. 2 in 2' and 1' erect perpendiculars from these points to cut the former line through H in C and K. Join the points as shown, and A B K G F' C is the hexagon required by Question 2. Lines from the points of the hexagon to the centre H divide it into six equal triangles.

(3) Draw a line from M. P. 1 through A to picture line as L; also lines from V. P. 1 through A and B. Set off on the picture line L M 5 ft., the length of the prism. Join M to M. P. 1, cutting the line from A in N—A N is one edge of the prism. Join N to V. P. 2, cutting the line from V. P. 1 in O. A B N O is the side of the prism on the ground. Draw a line from V. P. 1 through H: such line will contain the upper edge of the prism. Bring lines from M. P. 2 through N and O to the P. L. in N' and O'. Bisect N' O' in P'. Join P' to M. P. 2, cutting N O. Erect the vertical line, from where this line cuts N O, cutting the line from V. P. 1 through H in R'. Join R' N and R' O. Darken as shown.

UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATION, 1872.
LINEAR PERSPECTIVE

SCALE $\frac{1}{48}$

XLVII

LESSON XLVII.

UNIVERSITY OF CAMBRIDGE.—LOCAL EXAMINATIONS, 1872.

LINEAR PERSPECTIVE.

These Problems are to be worked to a scale of $\frac{1}{4}$ of an inch to the foot, the distance of the spectator from the picture being in each case, 14 ft., and the horizon 5 ft. above the ground plane. The lines by which each problem is solved must appear on the Paper when submitted to the Examiners.

Question 1.—A rectangular solid is 6 ft. long, and has a square base of 3 ft. sides. Put it into perspective placed upright on the ground plane with one side parallel to the picture. The angle nearest the spectator to be 1 ft. on his left, and 1 ft. within the picture.

Question 2.—Put into perspective a cube of 3 ft. edge resting on the ground plane with its vertical sides making angles of 45° with the picture plane. The nearest angle of the cube on the ground plane to be

3 ft. on the right of the spectator and 5 ft. within the picture.

Question 3.—From a point A in the picture line, 3 ft. on the right of the spectator, draw a line on the ground plane inclined towards the left at an angle of 30 degrees. From a point R on this line, 4 ft. from the picture line, draw a second line at right angles to the first.

Question 4.—Give a definition of vanishing points.

Scale, $\frac{1}{48}$.

Problem 1.—Obtain the P. S., P. D., H. L., P. L., from the instructions, as before explained.

This is a problem in parallel perspective, and presents no difficulty. Set off on P. L. point B 1 ft. to the left; join to P. S. Cut this line 1 ft. within the picture in B¹. B C is set off 3 ft., and B¹ C¹ drawn in perspective, as in Lesson IV. C D is the true height and C¹ D¹ the same height in perspective. Complete, as shown and explained, in the foregoing Lessons.

Problem 2. is a question in angular perspective of 45° .

Find a point 3 ft. to the right and 5 ft. within the picture (Lesson III.)

This is point E. The vanishing points for 45° are the points of distance. Obtain measuring points 1 and 2 as explained before for 45° ; join from E to V. P. 1 and V. P. 2. Bring lines from M. P. 1 and M. P. 2 to cut the picture line in G and H. Set off H 3 and G 5 each 3 ft. (side of cube); join to their respective measuring points, giving points J and K. Set up height A F =

3 ft.; join to P. S. in this case, because the height is set up from point A, from which a line is drawn to P. S.—E F¹ is the height in perspective. Complete as shown and explained in previous Lessons.

Problem 3.—Obtain the vanishing points for 30° on the left V. P. 3. Set off point A 3 ft. to the right, and draw a line from A to the vanishing point 3.

To find a point in this line 4 ft. from the picture line proceed thus:— Take any point X in P. L. Join to P. S. Set off X Y in P. L. 4 ft. Join to P. D., cutting line from X to P. S. in Z (Lesson III.) Draw Z Z¹ parallel to P. L. Every point in this line is 4 ft. from the picture line, it cuts line from A to V. P. for 30° in R. Obtain V. P. 4 for 60° , a line from R to this point is at right angles to the former line vanishing at 30° to the left.

Problem 4.—Vanishing point is that point in the horizon where lines, that are parallel to each other, appear to meet.

SCALE 48

EXAMINATION PAPER - MARCH 1880
SCIENCE & ART DEPARTMENT.

XLVIII

LESSON XLVIII.

EXAMINATION PAPER, MARCH 1873 & MARCH 1880.—SCIENCE AND ART DEPARTMENT.

Centre of vision is given; height of eye, 5 ft. 6 in.; distance of spectator from C. V. is 14 ft. by scale, which is $\frac{1}{4}$ in. to a foot ($\frac{1}{48}$).

N.B.— $\frac{1}{2}$ inch to a foot on the real paper.

(1) Line A B is the perspective representation of one edge of a cube, which rests upon its side on the ground plane. You are required to complete the representation of the cube. The lines used in working the Problem must be shown.

(2) C D is the perspective representation of one diameter of a circle on the ground plane. Complete the circle.

(3) Give the distance by scale from point C to the angle of the cube above point B.

(1) Obtain P.D.¹ and P.D.², etc., as before explained. Produce A B to cut the horizon in V.P. 1. Join V.P. 1 to S.P., and draw a line at right angles to the line from V.P. 1, cutting the horizon in V.P. 2. Obtain measuring point 1 and 2, as explained before. From M.P. 1 draw lines through A and B to cut the picture line in L K. L K is the true length of A B. Complete as shown.

(2) From P.D. 1 draw lines through C and D to cut the picture line in 1 and 2. Notice that C D produced meets the centre of vision. Then 1-2 is the true length of C D in perspective. Describe the semicircle as shown below the P.I., and complete the circle in perspective.

(3) Take points B and C out of perspective (B C¹), as explained in a previous Lesson. Join B C¹, and make the angle C¹ B B¹, 90°, and B B¹ as long as one edge of the cube. Join B¹ C¹: this line is the distance required in Question 3.

EXAMINATION PAPERS

SET BY THE

Science & Art Department, South Kensington,

From 1868 to 1880.

No. 1	<p><i>Examination Paper set Nov., 1868, and also in Nov., 1871.</i>—Find a point A on the picture line, 4 ft. to the left. Find the vanishing point for lines making angles of 60° with the picture plane, and draw from A to this vanishing point—on this line find points B and C, 3 ft. and 6 ft. from A respectively. BC is one side of an equilateral triangle; complete the triangle, and on it construct a prism 5 ft. high. Also give a definition of "Centre of Vision."</p>	No. 6	<p><i>Examination Paper set in 1868.</i>—The base of a rectangular solid is 3 ft. square, its length 8 ft. Put it in perspective lying on one of its 8 ft. sides, of which side the longer edges shall vanish at an angle of 45° with the picture plane towards the right. The nearest angle to be 3 ft. to the left and 1 ft. in the picture. Across the middle of this solid, and at right angles to it, place a similar one in a horizontal position. Let the centre of the upper solid be vertically over that of the lower one.</p>
No. 2	<p><i>Examination Paper set in Nov., 1868, and Nov., 1871.</i>—Place a circle in perspective, with its centre 3 ft. to the right and 4 ft. in the picture. Radius of the circle 3 ft.—on this base place a cylinder 7 ft. high.</p>	No. 7	<p><i>Examination Paper set in 1879.</i>—Find a point on the ground 3 ft. in the picture and 1 ft. to the left. Upon it erect a vertical line 6 ft. high. This line is a diagonal of a square, of which the horizontal diagonal makes an angle of 60° with the picture plane towards the left. Complete the perspective representation of the cube, of which this square forms a part. Also state what is understood by the term "Picture Plane."</p>
No. 3	<p><i>Examination Paper set in 1868 and 1871.</i>—The line AB is the perspective representation of one side of a regular hexagon lying on the ground. Give the perspective representation of a vertical prism 6 ft. high, standing on this hexagon as a base.</p>	No. 8	<p><i>Examination Paper set in 1869.</i>—A right cylinder—diameter 4 ft. and 4 ft. in length—the axis is vertical, and touches the ground plane 3 ft. in the picture and 1 ft. to the left. On this cylinder place a quadrangular pyramid 3 ft. high, the edges of whose square base make equal angles with the picture plane and are equidistant from the cylinder. One angle of the base touches the picture plane. Give the definition of a "Measuring Point."</p>
No. 5	<p><i>Examination Paper set in 1868.</i>—The base of a prism is an equilateral triangle of 5 ft. sides; its length is 8 ft. Put it into perspective with one side lying on the ground plane, its longer edges being inclined to the picture plane at angles of 60° to the right, and the angle nearest the spectator being 3 ft. to the left and 2 ft. in the picture. Put the same solid in perspective, standing on the ground plane in a vertical position. Two sides must be shown, and the third side be parallel to the picture plane. The nearest angle to be 1 ft. in the picture and 4 ft. on the right.</p>	No. 9	<p><i>Examination Paper set in 1869 and 1871.</i>—A horizontal line which is at an angle of 45° with the picture plane towards the right intersects the picture plane 4 ft. to the left of the spectator. Find points A C, D B upon this line respectively 2.4., 7.9. ft. from the point of intersection. The lines A C, D B are near edges of two solids which are square in plan, and 4 ft. high. From the top of these solids spring a semi-circular arch of the same thickness.</p>

No. 10 *Examination Paper set in 1869 and 1871.*—Find two points on the ground plane—one 3 ft. to the left and 7 ft. within, other 5 ft. to the left and 2 ft. within. A right line connecting these two points is the diagonal of the base of a cube standing on the ground plane. Complete the cube.

No. 11 *Examination Paper set in 1869 and 1871.*—A circle of 2 ft. radius is in the vertical plane at right angles to the picture plane at a point 4 ft. to the right and 3 ft. in the picture. Draw its perspective representation.

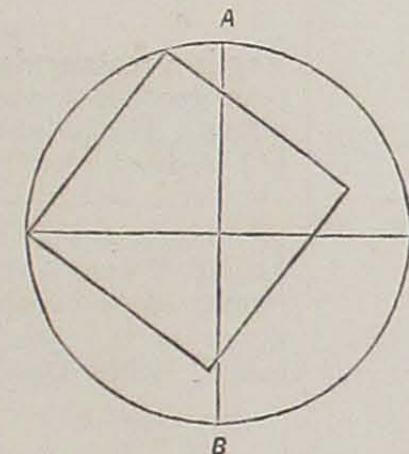
No. 12 *Examination Paper set in 1871.*—An octagonal prism 8 ft. long, its base having $1\frac{1}{2}$ ft. sides, and its end vanishing to the left at an angle of 30° with the picture plane, the nearest side of the end being 2 ft. to the left and 4 ft. within the picture.

No. 13 *Examination Paper set in 1871.*—Find two points 4 ft. to the right, one 1 ft. in the picture; the other 8 ft. in the picture. On the line between these points erect a semi-circular vertical arch.

No. 14 *Examination Paper set in May, 1875.*—Put this figure in perspective when B is 3 ft. to the left and 2 ft. from the picture plane, BA vanishing to the right at an angle of 45° with the picture plane.

Treat the square as end of prism whose axis is vertical and 3 ft. long. Complete it.

A point C is given on the ground within the picture. Find the length from C to A. Distance, 12 ft.; height of eye, 5 ft.; scale, $\frac{1}{4}$.

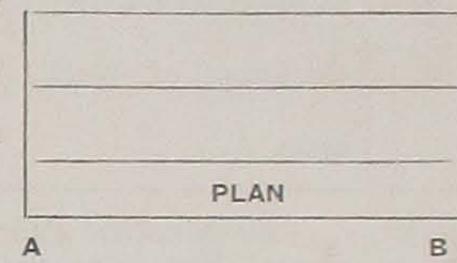
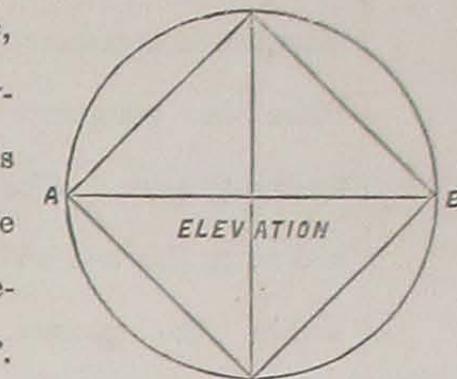


No. 15 *Examination Paper set in Training College, December, 1875.*—Distance of spectator, 12 ft.; eye, 5 ft. above ground plane.—Find a point A 4 ft. from ground plane. This point is centre of two circles of 7 ft. diameter. One circle lies in a vertical plane, vanishing at 50° to the right. Its lower edge touches the ground plane 4 ft. to the right. The other lies in a horizontal plane. Draw from A a line 9 ft. long, whose other extremity is in the ground plane.

No. 16 *Examination Paper set in December, 1875.*—Find a point 4 ft. to the left of the spectator and 5 ft. from picture plane; from that point erect a vertical line 6 ft. high. This line is the vertical height of a right pyramid whose base is an equilateral triangle of 6 ft. side. Give perspective representation of the solid.

No. 17 *Examination Paper set in May, 1876.*—Represent in perspective the cube and circular disc, shown half size, with the accompanying plan and elevation. The solids rest upon the ground plane, and the diagonal A B is horizontal and recedes towards the right hand at 50° .

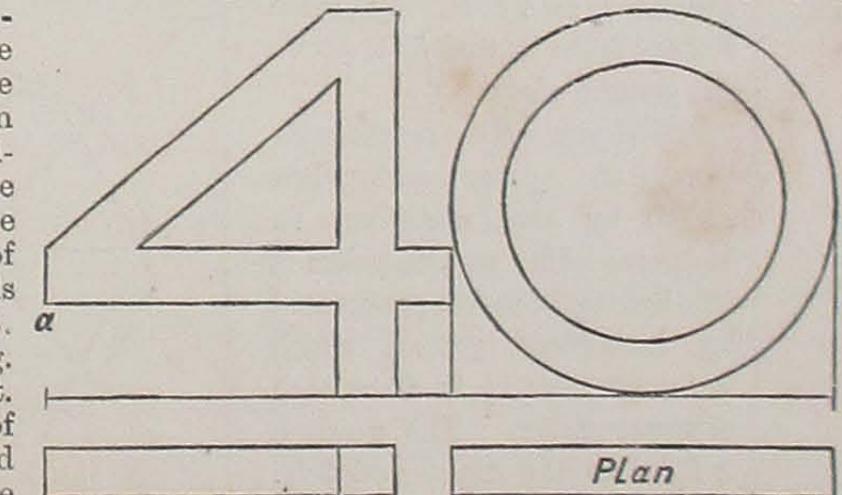
The angle A touches the picture plane at a point vertically below the *centre of vision*. The centre of vision is given. The eye of spectator is to be 12 ft. (by scale), "which is $\frac{1}{2}$ -inch to a foot" distant from it, and 5 ft. above the ground plane.



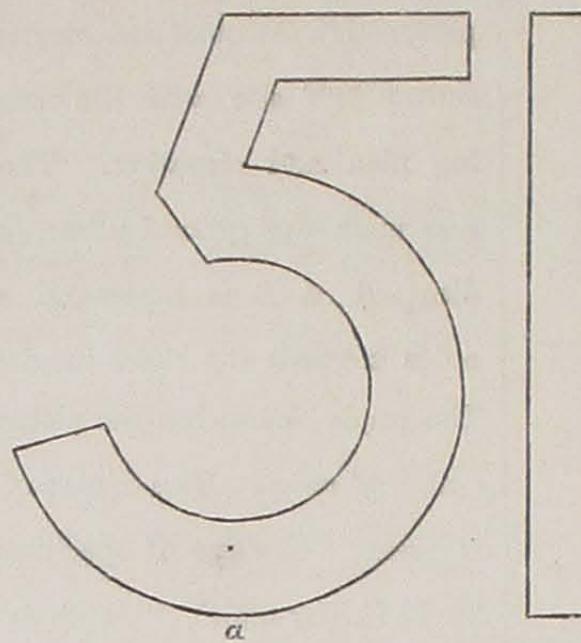
No. 18 *Examination Paper set in May, 1876.*—A cube, 5 ft. edge, stands upon the ground plane on one of its faces, and its horizontal edges make angles of 40° and 50° with the picture plane. The nearest point on the ground plane is 3 ft. from the picture line, and 2 ft. from the spectator's right.

A square slab of 8 ft. side and 2 ft. thick rests upon the upper face of the cube, its edges projecting equally on all sides and its vertical faces parallel to the vertical faces of the cube. Give the perspective projection of these two blocks. Centre of vision is given; the distance is 12 ft., and height of eye 5 ft.

No. 19 *Examination Paper set in May, 1877.*—Give the perspective representation of the two figures shown half size in the accompanying plan and elevation. The faces of the figures lie in planes receding from the picture plane at an angle of 35° towards the right. Point a in fig. 4 is to be 1 ft. on the left of spectator and 2 ft. within the picture. Centre of vision given; distance, 12 ft. height of eye, 5 ft.



No. 20 *Examination Paper set in May, 1877.*—Height of eye, 5 ft.; distance of spectator, 12 ft. Place in perspective the figures in plan. The point *a* 3 ft. to the left and 4 ft. within the picture, and vanishing at an angle of 40° to the right standing vertical; thickness of block, 1 ft.; scale, $\frac{1}{4}$.

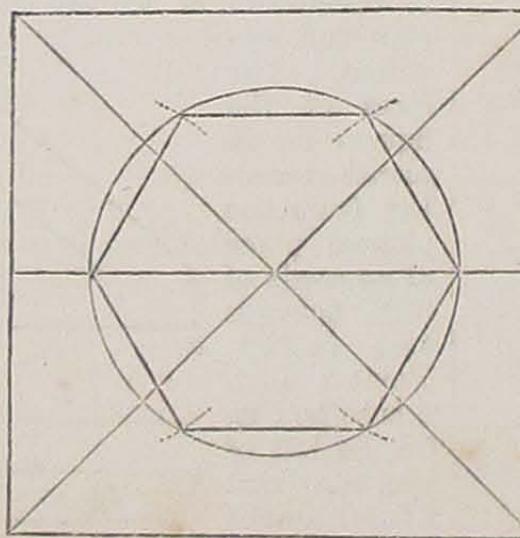


No. 21 *Examination Paper set in March, 1878.*—Centre of Vision is given. Eye of spectator is 12 ft. by scale distant from it, and 5 ft. above the ground plane.

- (1) Give the perspective representation upon the G. P. of a circle of 3 ft. radius, which touches the picture plane at a point 1 ft. to the left of spectator.
- (2) Hexagonal pyramid of 2 ft. edge at base and 6 ft. high, stands in the centre of this circle. Two edges of its base are parallel to picture plane. Give its perspective projection.
- (3) A point A is upon the ground plane. Find its true distance from the apex of the pyramid; show lines in working.

No. 22 *Examination Paper set in May, 1879.*—DIRECTIONS—The centre of vision is given. The eye of spectator is 12 ft. by scale distant from it, and 5 ft. above the ground plane.

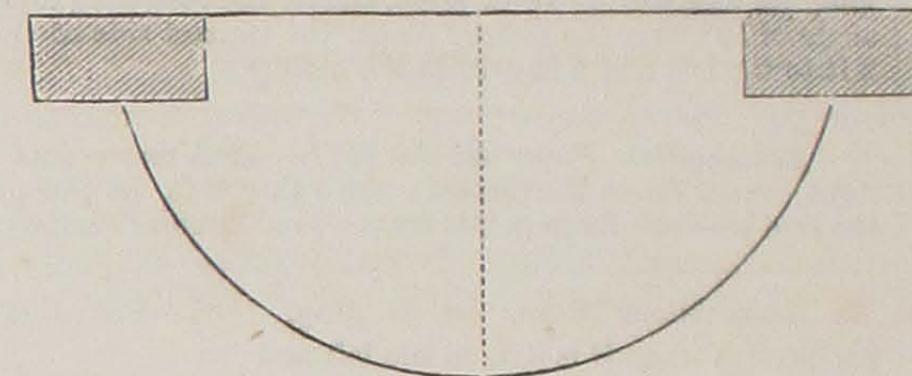
Give perspective representation of the square right prism shown by end elevation in diagram. The square prism is 3 ft. in length, and is penetrated by hexagonal prism, which latter projects $1\frac{1}{2}$ ft. from each of square faces. The point *a* is 2 ft. to the right and 2 ft. within, the axis of solids vanish to left at 55° .



No. 23 (1) *Examination Paper set in March, 1878.*—The base of a rectangular solid is 4 ft. square; its length 8 ft. Put it in perspective lying on one of its 8 ft. sides, of which side the longest edges shall vanish at an angle of 50° towards the right hand. The angle nearest the picture to be 3 ft. to the left of spectator and 1 ft. within the picture.

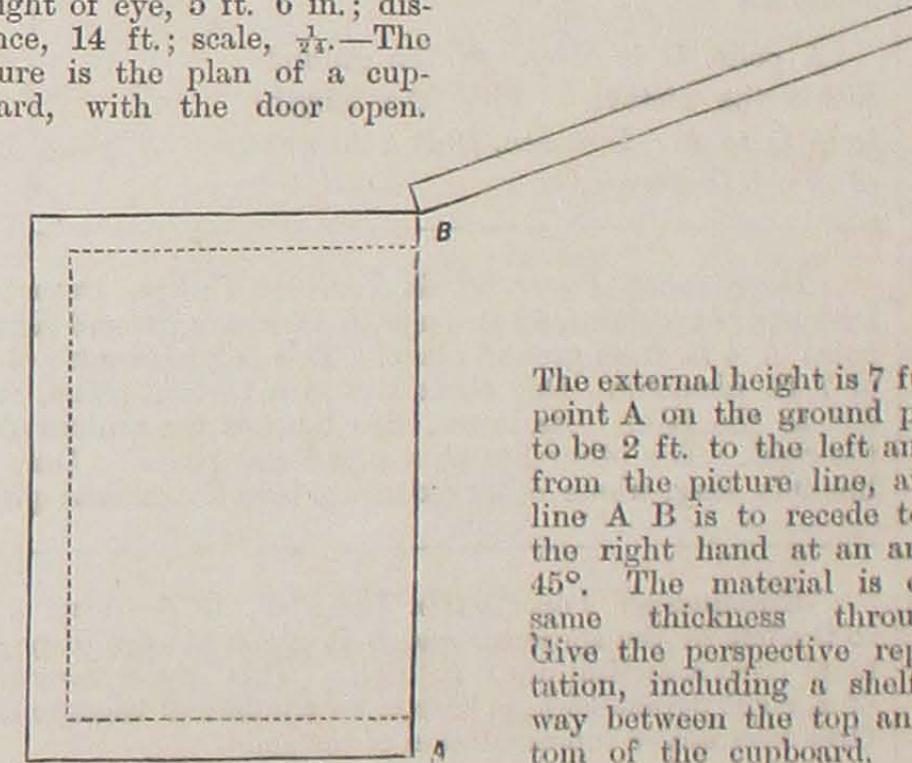
(2) Give the distance, according to scale, of each visible angle of the solid from the plane of the picture. Centre of vision given. The distance is 12 ft. and height of eye 5 ft.

No. 24 *Examination Paper set in May, 1880.*—Centre of vision is given on the paper; height of eye, 5 ft. 6 in.; distance, 14 ft.; scale, $\frac{1}{4}$ ($\frac{1}{2}$ in. represents 1 ft.)—The figure is the plan of a doorway through a wall, with semicircular step. The step is 1 ft. thick, and the door-



way is a rectangular opening 8 ft. in height. The dotted line in the plan recedes towards the right at an angle of 50° to the picture plane. The point where the dotted line meets the semicircular edge is 1 ft. to the left and 4 ft. within the picture.

No. 25 *Examination Paper set in May, 1880.*—Centre of vision is given; height of eye, 5 ft. 6 in.; distance, 14 ft.; scale, $\frac{1}{4}$.—The figure is the plan of a cupboard, with the door open.



The external height is 7 ft. The point A on the ground plane is to be 2 ft. to the left and 2 ft. from the picture line, and the line A B is to recede towards the right hand at an angle of 45° . The material is of the same thickness throughout. Give the perspective representation, including a shelf half-way between the top and bottom of the cupboard.

No. 26 *Examination Paper set in November, 1878.*—The centre of vision is given; height of eye, 5 ft. by scale, which is $\frac{1}{2}$ in. to a foot (or $\frac{1}{4}$); distance of spectator, 12 ft.

(1) The axis of a right cylinder, 6 ft. long and 4 ft. in diameter, is 2 ft. to the left and 3 ft. within the picture plane. Place it in perspective.

(2) From a point, 2 ft. to the right and 1 ft. within the picture, draw a line in the same plane as the axis of the cylinder, and cutting the top of the cylinder.

NOTES BY THE AUTHOR OF THE TEXT BOOK.—To work this Problem, let the student find the plans of two points (Quest. 1 and 2). The former 2 ft. to the left and 3 ft. below the picture line; the other 2 ft. to the right and 1 ft. below the picture line. Join these points and produce the line; it contains the axis of the cylinder. Draw the plan of the cylinder on the axis and put it in perspective either by angular or parallel perspective.

No. 27 *Examination Paper set in March, 1879.*—Centre of vision given; distance, 12 ft.; height of eye, 6 ft.

(1) Five slabs, each 1 ft. thick, are to be shown placed as though they formed a rectangular box, without a lid, standing on the ground plane; outside measurements of which are—8 ft. long, 6 ft. broad, and 3 ft. high. Nearest angle, 7 ft. to left and 4 ft. within. The long edges to vanish to the right at 45° .

(2) A Point C, in perspective on the right, is the centre of a circle on the ground plane, of which the circumference touches the picture line. Draw the circle in perspective.

(3) On this circle, as base, complete a cylinder 7 ft. in height.

PERSPECTIVE. TIME ALLOWED, }
HALF-AN-HOUR.

H. M. S. BRITANNIA, DECEMBER, 1875.

INSTRUCTION.

Let the distance of the spectator from the picture be 20 ft. and the horizon 8 ft. above the ground plane. Scale $\frac{1}{4}$ inch to the foot. The lines by which the problem is worked must be shown.

No. 1 **QUESTION.—FIRST TERM.**

Put into perspective a cube of which the edges are 10 ft. long. Let one face of the cube be on the ground plane, and one visible face parallel to the picture. Let the nearest corner of the cube be 3 ft. on left of the spectator, and 2 ft. from the plane of the picture.

No. 2 **QUESTION.—SECOND TERM.**

Place a cube of 6 ft. edge on the ground plane, 6 ft. on the left of the spectator, and one of its vertical edges touch the picture. Let its vertical faces be at angles of 45° with the picture plane.

No. 3 **QUESTION.—THIRD TERM.**

Put into perspective a regular hexagonal prism of which the sides of the base are 4 ft. and the height 10 ft. Let it stand on its base on the ground plane, with its nearest corner 2 ft. on the left of the spectator and 4 feet from the picture line, and let two of its vertical faces be parallel with the picture.

No. 4 **QUESTION.—FINAL EXAMINATION.**

Put into perspective a regular prism 8 ft. long and having an equilateral triangle of 5 ft. sides as a base. Let it lie with one face on the ground plane, with its long edges vanishing towards the right hand at 30° with the picture, and let its nearest corner touch the picture 2 ft. on the right of the spectator.

PRACTICAL PERSPECTIVE PAPERS.

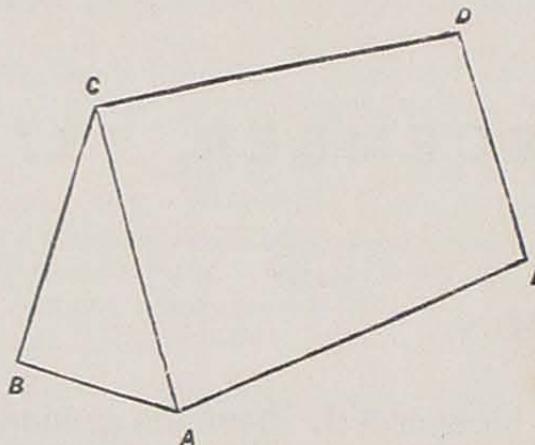
Civil Service Commission—Military Examinations.

SANDHURST.

No. 1 A B C D E (see *Figure*) is the perspective of the middle one of three contiguous and similar gable ends.

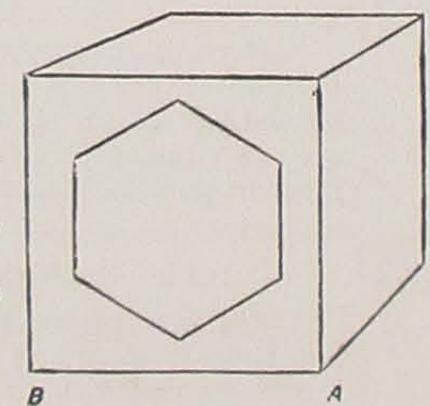
Draw the other two.

NOTE.—Find the horizon and the vanishing line of the plane of the gables, and in that line the vanishing points of B A and A E.



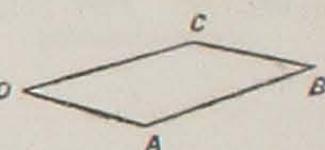
No. 2 A B C is the perspective of a cube with a hexagon on the front face; transfer the hexagon in perspective to the side face A C.

NOTE.—Leave your lines of construction in pencil. The neatness of these lines, as well as those of the hexagon, is very important.



ROYAL ACADEMY, WOOLWICH.

No. 1 A B C D is the perspective outline of a square; inscribe in it another square, whose sides are parallel to those of A B C D and half their size, and D having the same centre.



No. 2 Draw in perspective three equal cubes, placed so as to stand in the same line with spaces between them equal to the cubes. The three cubes are to recede from the spectator at an angle of 45° towards the right, so that the two near faces of each cube make equal angles with the picture. Edge of nearest cube, 4 in.

No. 3 Draw in perspective a small oblong table with a chair before it, both standing at an angle of 45° to the spectator's line of vision.

The Author's Test Papers on Parallel Perspective.

TO BE WORKED BY THE STUDENT.

Height of Eye, 5 ft.; distance, 14 ft.; scale, $(\frac{1}{24})$ $\frac{1}{2}$ in. to 1 ft.

No. 1 (1) *On the right.*—Find a point A on the ground 4 ft. to the right and 3 ft. 6 in. within the picture. From the point A draw a line A B parallel to the picture plane, 5 ft. long. From the centre of the line and receding from the picture plane, draw a line at right angles to A B, cut off 2 ft. of this line.

(2) *On the left.*—The centre of a circle on the ground is 3 ft. 6 in. to the left and 2 ft. within the picture. The circle touches the picture plane. Place it in perspective.

No. 2 (1) Find two points, A and B, on the ground ; A 5 ft. to the left and B 3 ft. to the right of the spectator, each point being 2 ft. within the picture.

Join A B. The line A B is the edge of a cube resting on the ground, with a face parallel to the picture plane. Complete the perspective projection of the cube, showing by dotted lines all the edges not visible to the spectator.

No. 3 (1) *On the left.*—Find two points, A and B, in the picture plane ; A 3 ft. and B 5 ft. to the spectator's left. A B is the side of a hexagon lying on the ground ; place it in perspective. Suppose this hexagon to be the base of a hexagonal pyramid 7 ft. high. Complete the perspective projection of it.

On the right.—Represent in perspective a stick, 5 ft. high, penetrating the ground, and at right angles to it, at a point 6 ft. to the right and 5 ft. within the picture.

(3) What length string will reach from the top of this stick to the top of the pyramid ?

No. 4 (1) *On the left.*—Two points A and B on the ground are each 4 ft. 6 in. to the left. A is 3 ft. within the picture and B 6 ft. The line A B is a diameter of a circle lying on the ground. Complete the perspective representation of it. Suppose this circle to be the end of a cylinder 8 ft. long, its axis perpendicular to the ground. Complete it.

On the right.—Place this cylinder in perspective on the right, lying on the ground, with its axis parallel to the picture plane and its nearest end 3 ft. 6 in. to the right, the cylinder touching the picture plane.

No. 5 (1) *On the right.*—A block of wood, 5 ft. square and 1 ft. thick, lies on the ground on one of its square faces, one side parallel to the picture plane ; its nearest angle 3 ft. to the right and 2 ft. within the picture. Suppose the top of this to be the base of a square pyramid 6 ft. high. Complete the perspective projection of it.

(2) *On the left.*—The axis of a pentagonal prism enters the ground at a point 6 ft. to the left and 5 ft. within the picture. One face is parallel to the picture plane, and each face measures 8 ft. by 2 ft. Place it in perspective.

(3) What length string will reach from the centre of the base of the pentagonal prism to the top of the pyramid.

No. 6 (1) *On the right.*—Put the following points in perspective : A 3 ft. to the right and B 6 ft. to the right ; A and B each one foot within the picture ; C 8 ft. to the right and 3 ft. within the picture ; D 9 ft. 6 in. to the right and 2 ft. within the picture. Join A B, B C, and C D. This is the plan of a folding screen 8 ft. high. Complete the perspective projection of it.

(2) *On the left.*—Show in perspective a flight of six steps, of 3 in. tread and 6 in. rise, with the end parallel to the picture plane, and nearest angle of the first step 3 ft. to the left and 2 ft. within the picture plane.

NOTE.—MR. ANDREWS, Charles' Schools, Plymouth, will send a correct solution, or correct the papers of private students of any exercise or test paper in this book, post free for 4d. each paper, in stamps.

The Author's Test Papers on Angular Perspective.

TO BE WORKED BY THE STUDENT.

Height of Eye, 5 ft. ; distance, 14 ft. ; scale, $\frac{1}{24}$ ($\frac{1}{2}$ in. to 1 ft.)

No. 1	(1) Find a point 3 ft. to the left and 1 foot within the picture. From this point draw a line 7 ft. long, vanishing at an angle of 30° towards the left ; and from the end of this line, farthest from the picture plane, draw a line at right angles to the former, and vanishing to the right. Define vanishing points, measuring points, and centre of vision.	No. 4	(1) Describe a circle 2 in. in diameter (full-size scale), and from the same centre describe another circle $1\frac{1}{4}$ in. in diameter. Suppose this to be the end of an iron pipe 8 ft. long ; place it in perspective, its axis making an angle of 60° towards the right with the picture plane, and the pipe touching that plane directly opposite the eye.
No. 2	(1) The axis of an octagonal prism vanishes towards the right at an angle of 40° . Each side of the base is 1 ft. long. The nearest angle touches the picture plane 1 ft. to the right. The prism rests on one face on the ground. Give the perspective projection of it.	No. 5	Represent in perspective a box 8 ft. by 6 ft. and 4 ft. deep (outside measurements). The cover is 1 ft. thick, and thickness of all material is 6 inches. The nearest corner to be 3 ft. to the left and 2 ft. within the picture plane, the longer edges vanishing to the right at an angle of 45° , and the cover of the box open at an angle of 45° .
No. 3	(1) Place the octagonal prism, given in the previous paper, in perspective when its axis enters the ground 3 ft. to the left and 5 ft. within the picture, one of its faces making an angle of 50° on the left with the picture plane. (2) What length of string will reach from a point 6 ft. to the right and 1 ft. within the picture to the nearest angle of the top of the prism.	No. 6	Two circles intersect one another at right angles. Their centres coincide. Represent them in perspective when one circle is in a vertical plane, making an angle on the right of 35° with the picture plane, and their common centre is 5 ft. to the left and 6 ft. within the picture. The diameter of each circle is 6 ft.

NOTE.—Mr. ANDREWS, Charles' Schools, Plymouth, will send a correct solution, or correct papers of private students of any exercise or test paper in this book, post free, for 4d. each paper, in stamps.

THE END

